

Chapter 1 Direct and Inverse Proportion

TEACHING NOTES

Suggested Approach

In Secondary One, students have learnt rates such as \$0.25 per egg, or 13.5 km per litre of petrol etc. Teachers may wish to expand this further by asking what the prices of 2, 4 or 10 eggs are, or the distance that can be covered with 2, 4 or 10 litres of petrol, and leading to the introduction of direct proportion. After students are familiar with direct proportion, teachers can show the opposite scenario that is inverse proportions.

Section 1.1: Direct Proportion

When introducing direct proportion, rates need not be stated explicitly. Rates can be used implicitly (see Investigation: Direct Proportion). By showing how one quantity increases proportionally with the other quantity, the concept should be easily relatable. More examples of direct proportion should be discussed and explored to test and enhance thinking and analysis skills (see Class Discussion: Real-Life Examples of Quantities in Direct Proportion). Teachers should discuss the linkages between direct proportion, algebra, rates and ratios to assess and improve students' understanding at this stage (see page 4 of the textbook). Teachers should also show the unitary method and proportion method in the worked example and advise students to adopt the method that is most comfortable for them.

Section 1.2: Algebraic and Graphical Representations of Direct Proportion

By recapping what was covered in the previous section, teachers should easily state the direct proportion formula between two quantities and the constant k . It is important to highlight the condition $k \neq 0$ as the relation would not hold if $k = 0$ (see Thinking Time on page 6).

Through studying how direct proportion means graphically (see Investigation: Graphical Representation of Direct Proportion), students will gain an understanding on how direct proportion and linear functions are related, particularly the positive gradient of the straight line and the graph passing through the origin. The graphical representation will act as a test to determine if two variables are directly proportional.

Section 1.3: Other Forms of Direct Proportion

Direct proportion does not always involve two linear variables. If one variable divided by another gives a constant, then the two variables are directly proportional (see Investigation: Other Forms of Direct Proportion). In this case, although the graph of y against x will be a hyperbola, the graph of one variable against the other will be a straight line passing through the origin. Teachers may wish to illustrate the direct proportionality clearly by replacing variables with Y and X and showing $Y = kX$, which is in the form students learnt in the previous section.

Section 1.4: Inverse Proportion

The other form of proportion, inverse proportion, can be explored and studied by students (see Investigation: Inverse Proportion). When one variable increases, the other variable decreases proportionally. It is the main difference between direct and inverse proportion and must be emphasised clearly.

Students should be tasked with giving real-life examples of inverse proportion and explaining how they are inversely proportional (see Class Discussion: Real-Life Examples of Quantities in Inverse Proportion).

Teachers should present another difference between both kinds of proportions by reminding students that $\frac{y}{x}$ is a constant in direct proportion while xy is a constant in inverse proportion (see page 20 of the textbook).

Section 1.5: Algebraic and Graphical Representations of Inverse Proportion

Similar to direct proportion teachers can write the inverse proportion formula between two quantities and the constant k . It is important to highlight the condition $k \neq 0$ as the relation would not hold if $k = 0$ (see Thinking Time on page 23).

Although plotting y against x gives a hyperbola, and does not provide any useful information, teachers can show by plotting y against $\frac{1}{x}$ and showing direct proportionality between the two variables (see Investigation: Graphical Representation of Inverse Proportion).

Section 1.6: Other Forms of Inverse Proportion

Inverse proportion, just like direct proportion, may not involve two linear variables all the time. Again, teachers can replace the variables with Y and X and show the inverse proportionality relation $Y = \frac{k}{x}$.

Challenge Yourself

Question 1 involves stating the relations between the variables algebraically and manipulating the equations. For Question 2, teachers may wish to state the equation relating z , x^2 and $\sqrt{y}$ outright initially, and explain that when z is directly proportional to x^2 , $\sqrt{y}$ is treated as a constant. The same applies to when z is inversely proportional to $\sqrt{y}$, x^2 will be considered as a constant. Question 3 follows similarly from Question 2.

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WORKED SOLUTIONS

Investigation (Direct Proportion)

1. The fine will increase if the number of days a book is overdue increases.

$$\begin{aligned}\frac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 3 days}} &= \frac{90}{45} \\ &= 2\end{aligned}$$

The fine will be doubled if the number of days a book is overdue is doubled.

$$\begin{aligned}\frac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 2 days}} &= \frac{90}{30} \\ &= 3\end{aligned}$$

The fine will be tripled if the number of days a book is overdue is tripled.

$$\begin{aligned}\frac{\text{Fine when a book is overdue for 5 days}}{\text{Fine when a book is overdue for 10 days}} &= \frac{75}{150} \\ &= \frac{1}{2}\end{aligned}$$

The fine will be halved if the number of days a book is overdue is halved.

$$\begin{aligned}\frac{\text{Fine when a book is overdue for 3 days}}{\text{Fine when a book is overdue for 9 days}} &= \frac{45}{135} \\ &= \frac{1}{3}\end{aligned}$$

The fine will be reduced to $\frac{1}{3}$ of the original number if the number

of days a book is overdue is reduced to $\frac{1}{3}$ of the original number.

Class Discussion (Real-Life Examples of Quantities in Direct Proportion)

The following are some real-life examples of quantities that are in direct proportion and why they are directly proportional to each other.

- In an hourly-rated job, one gets paid by the number of hours he worked. The longer one works, the more wages he will get.
The wages one gets is directly proportional to the number of hours he worked.
- A Singapore dollar coin weights approximately 6 g. As the number of coins increases, the total mass of the coins will increase proportionally. The total mass of the coins is directly proportional to the number of coins.
- The circumference of a circle is equivalent to the product of π and the diameter of the circle. As the diameter increases, the circumference increases proportionally. The circumference of the circle is directly proportional to the diameter of the circle.
- The speed of a moving object is the distance travelled by the object per unit time. If the object is moving at a constant speed, as the distance travelled increases, then the time spent in travelling increases proportionally. The distance travelled by the object is directly proportional to the time spent in travelling for an object moving at constant speed.

- The length of a spring can be compressed or extended depending on the force applied on it. The force required to compress or extend a spring is directly proportional to the change in the length of the spring. This is known as Hooke's Law, which has many practical applications in science and engineering.

Teachers may wish to note that the list is not exhaustive.

Thinking Time (Page 6)

If we substitute $k = 0$ into $y = kx$, then $y = 0$.

This implies that for all values of x , $y = 0$.

y cannot be directly proportional to x in this case.

Investigation (Graphical Representation of Direct Proportion)

$y = 15x$ in this context means that for any additional number of a day a book is overdue, the fine will increase by 15 cents.

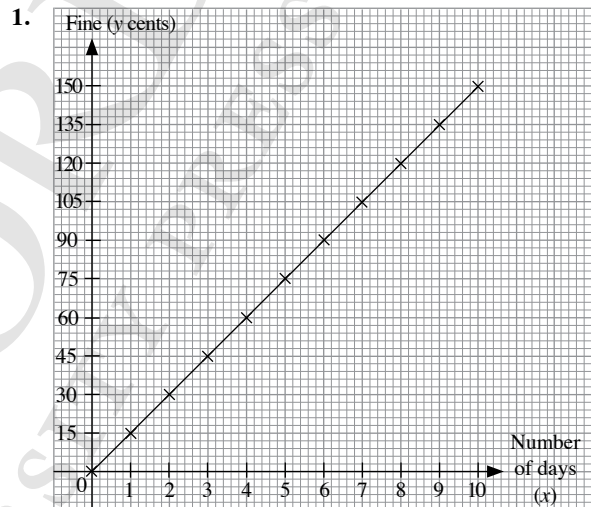


Fig. 1.1

2. The graph is a straight line.
3. The graph passes through the origin.

Thinking Time (Page 7)

1. Since y is directly proportional to x , $y = kx$

$$x = \left(\frac{1}{k}\right)y$$

Since $k \neq 0$, then we can rename $\frac{1}{k} = k_1$ where k_1 is another constant.

Hence, $x = k_1y$, where $k_1 \neq 0$ and x is directly proportional to y .

2. $x = k_1y$ is the equation of a straight line. When $y = 0$, $x = 0$.
We will get a straight line of x against y that passes through the origin.
3. If the graph of y does not pass through the origin, then $y = kx + c$, when $c \neq 0$. Since x and y are not related in the form $y = kx$, y is not directly proportional to x .
4. As x increases, y also increases. This does not necessarily conclude that y is directly proportional to x . It is important that when x increases, y increases **proportionally**. Also, when $x = 0$, $y = 0$.
 $y = kx + c$ is an example of how x increases and y increases, but y is not directly proportional to x .

Investigation (Other Forms of Direct Proportion)

1. y is not directly proportional to x . The graph of y against x is not a straight line that passes through the origin.

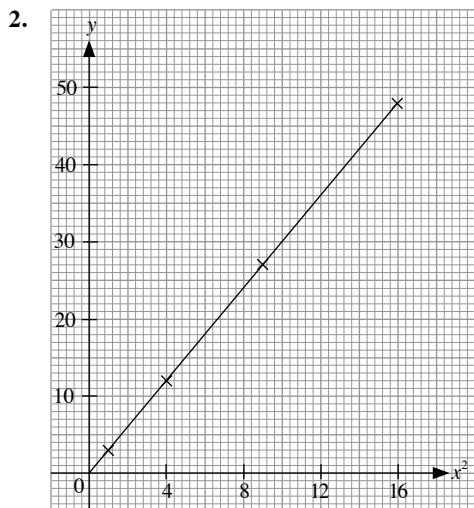


Fig. 1.3

y is directly proportional to x^2 . The graph of y against x^2 is a straight line that passes through the origin.

Thinking Time (Page 15)

1. $C = 0.2n + 20$

$$C - 20 = 0.2n$$

$$\frac{C - 20}{n} = 0.2$$

$$\frac{n}{C - 20} = 5$$

Since $\frac{n}{C - 20} = 5$ is a constant, then n is directly proportional to $C - 20$.

The variable is $C - 20$.

2. $y - 1 = 4x$

$$\frac{y - 1}{x} = 4$$

Since $\frac{y - 1}{x} = 4$ is a constant, then $y - 1$ is directly proportional to x .

Investigation (Inverse Proportion)

1. The time taken decreases when the speed of the car increases.

$$\begin{aligned} 2. \quad \frac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 20 km/h}} &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

The time taken will be halved when the speed of the car is doubled.

$$\begin{aligned} 3. \quad \frac{\text{Time taken when speed of the car is 60 km/h}}{\text{Time taken when speed of the car is 20 km/h}} &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

The time taken will be reduced to $\frac{1}{3}$ of the original number when the speed of the car is tripled.

$$\begin{aligned} 4. \quad \frac{\text{Time taken when speed of the car is 30 km/h}}{\text{Time taken when speed of the car is 60 km/h}} &= \frac{4}{2} \\ &= 2 \end{aligned}$$

The time taken will be doubled when the speed of the car is halved.

$$\begin{aligned} 5. \quad \frac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 120 km/h}} &= \frac{3}{1} \\ &= 3 \end{aligned}$$

The time taken will be tripled when the speed of the car is reduced to $\frac{1}{3}$ of its original speed.

Class Discussion (Real-Life Examples of Quantities in Inverse Proportion)

The following are some real-life examples of quantities that are in inverse proportion and why they are inversely proportional to each other.

- Soldiers often dig trenches while serving in the army. The more soldiers there are digging the same trench, the faster it will take. The time to dig a trench is therefore inversely proportional to the number of soldiers.
- The area of a rectangle is the product of its length and breadth. Given a rectangle with a fixed area, if the length increases, then the breadth decreases proportionally. Therefore, the length of the rectangle is inversely proportional to the breadth of the rectangle.
- The density of a material is the mass of the material per unit volume. For an object of a material with a fixed mass, the density increases when the volume decreases proportionally. The density of the material is inversely proportional to the volume of the material.
- The speed of a moving object is the distance travelled by the object per unit time. For the same distance, when the speed of the object increases, the time to cover the distance is decreased proportionally. The speed of the object is inversely proportional to the time to cover a fixed distance.
- For a fixed amount of force applied on it, the acceleration of the object is dependent on the mass of the object. When the mass of the object increases or decreases, the acceleration of the object decreases or increases proportionally. This is known as Newton's Second Law and has helped to explain many physical phenomena occurring around us.

Teachers may wish to note that the list is not exhaustive.

Thinking Time (Page 23)

If we substitute $k = 0$ into $y = \frac{k}{x}$, then $y = 0$.

This implies that for all values of x , $y = 0$

y cannot be inversely proportional to x in this case.

Investigation (Graphical Representation of Inverse Proportion)

1. We would obtain a graph of a hyperbola.
- 2.

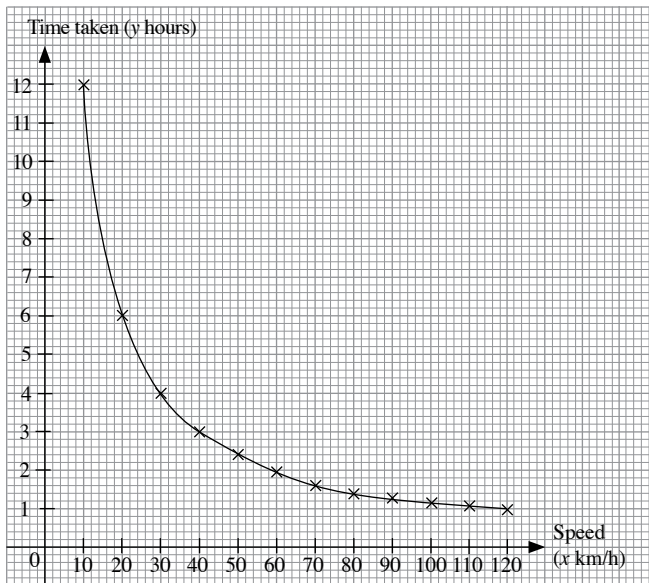


Fig. 1.4

3. When $x = 20$, $y = 6$.
When $x = 40$, $y = 3$.

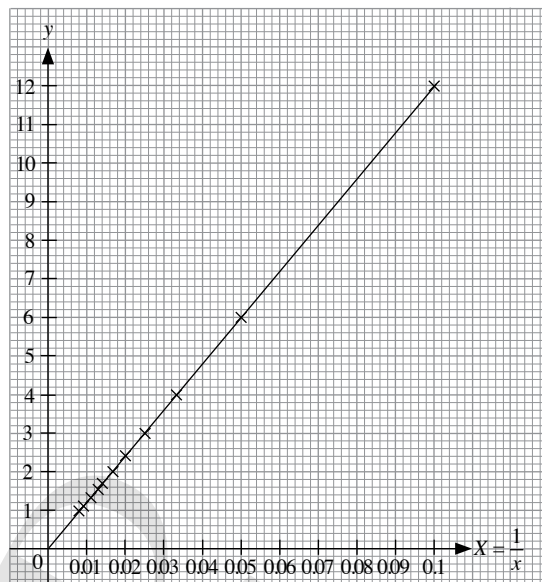
$$\begin{aligned} \text{Change in value of } y &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

The value of y will be halved when the value of x is doubled.

Speed (x km/h)	10	20	30	40	50	60
$X = \frac{1}{x}$	0.1	0.05	0.033	0.025	0.02	0.017
Time taken (y hours)	12	6	4	3	2.4	2

Speed (x km/h)	70	80	90	100	110	120
$X = \frac{1}{x}$	0.014	0.013	0.011	0.01	0.009	0.008
Time taken (y hours)	1.7	1.5	1.3	1.2	1.1	1

4.



5. The graph is a straight line that passes through the origin.
 $\frac{y}{X}$ is a constant.
6. y is directly proportional to X .
7. $y = kX$, where k is a constant. $\frac{y}{X}$ is a constant and y is directly proportional to X .

Thinking Time (Page 26)

Since y is inversely proportional to x , $y = \frac{k}{x}$
 $x = \frac{k}{y}$

Hence, $x = \frac{k}{y}$, where $k \neq 0$ and x is inversely proportional to y .

Practise Now 1

- The cost of the sweets is directly proportional to the mass of the sweets.

Method 1: Unitary Method

50 g of sweets cost \$2.10.

1 g of sweets cost $\frac{\$2.10}{50}$.

380 g of sweets cost $\frac{\$2.10}{50} \times 380 = \$15.96$.

\$15.96 = \$15.95 (to the nearest 5 cents)

Method 2: Proportion Method

Let the cost of 380 g of sweets be \$ x .

$$\text{Then } \frac{x}{380} = \frac{2.1}{50} \cdot \left(\frac{x_1}{y_1} = \frac{x_2}{y_2} \right)$$

$$x = \frac{2.1}{50} \times 380$$

$$= 15.96$$

$$= 15.95 \text{ (to the nearest 5 cents)}$$

Alternatively,

$$\frac{x}{2.1} = \frac{380}{50} \left(\frac{x_1}{x_2} = \frac{y_1}{y_2} \right)$$

$$x = \frac{380}{50} \times 2.1$$

$$= 15.96$$

$$= 15.95 \text{ (to the nearest 5 cents)}$$

$\therefore$ 380 g of sweets cost \$15.95.

(b) The amount of metal is directly proportional to the mass of the metal.

Method 1: Unitary Method

$\frac{3}{4}$ of a piece of metal weighs 15 kg.

A whole piece of metal weighs $\frac{15}{\frac{3}{4}}$ kg.

$\frac{2}{5}$ of a piece of metal weighs $\frac{15}{\frac{3}{4}} \times \frac{2}{5} = 8$ kg.

Method 2: Proportion Method

Let the mass of $\frac{2}{5}$ of the piece of metal be x kg.

$$\text{Then } \frac{x}{\frac{2}{5}} = \frac{15}{\frac{3}{4}} \left(\frac{x_1}{y_1} = \frac{x_2}{y_2} \right)$$

$$x = \frac{15}{\frac{3}{4}} \times \frac{2}{5}$$

$$= 8$$

Alternatively,

$$\frac{x}{15} = \frac{\frac{2}{5}}{\frac{3}{4}} \left(\frac{x_1}{x_2} = \frac{y_1}{y_2} \right)$$

$$x = \frac{\frac{2}{5}}{\frac{3}{4}} \times 15$$

$$= 8$$

$\therefore$ The mass of $\frac{2}{5}$ of the piece of metal is 8 kg.

Practise Now 2

1. (i) Since y is directly proportional to x ,
then $y = kx$, where k is a constant.

When $x = 2$, $y = 10$,

$$10 = k \times 2$$

$$\therefore k = 5$$

$$\therefore y = 5x$$

- (ii) When $x = 10$,

$$y = 5 \times 10$$

$$= 50$$

Alternatively,

when $x = 10$, (x increased by 5 times)

$y = 5 \times 10$ (y increased by 5 times)

$$= 50$$

We can also use $\frac{y_2}{y_1} = \frac{x_2}{x_1}$,

$$\text{i.e. } \frac{y}{10} = \frac{10}{2}$$

$$y = 5 \times 10$$

$$= 50$$

- (iii) When $y = 60$,

$$60 = 5x$$

$$\therefore x = \frac{60}{5}$$

$$= 12$$

2. Since y is directly proportional to x ,

$$\frac{y_2}{y_1} = \frac{x_2}{x_1}$$

$$\frac{y}{5} = \frac{7}{2}$$

$$y = \frac{7}{2} \times 5$$

$$= 17.5$$

- 3.

x	4	5	7	8	9.5
y	24	30	42	48	57

Since y is directly proportional to x ,

then $y = kx$, where k is a constant.

When $x = 5$, $y = 30$,

$$30 = k \times 5$$

$$\therefore k = 6$$

$$\therefore y = 6x$$

When $y = 48$,

$$48 = 6 \times x$$

$$x = \frac{48}{6}$$

$$= 8$$

When $y = 57$,

$$57 = 6 \times x$$

$$x = \frac{57}{6}$$

$$= 9.5$$

When $x = 4$,

$$y = 6 \times 4$$

$$= 24$$

When $x = 7$,

$$y = 6 \times 7$$

$$= 42$$

Practise Now 3

- (i) Since C is directly proportional to d ,
then $C = kd$, where k is a constant.

When $d = 60$, $C = 100$,

$$100 = k \times 60$$

$$\therefore k = \frac{5}{3}$$

$$\therefore C = \frac{5}{3}d$$

- (ii) When $d = 45$,

$$C = \frac{5}{3} \times 45 \\ = 75$$

$\therefore$ The cost of transporting goods is \$75.

- (iii) When $C = 120$,

$$120 = \frac{5}{3} \times d$$

$$d = 120 \times \frac{3}{5}$$

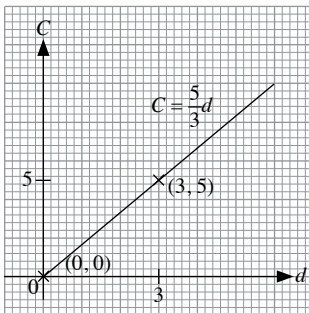
$$= 72$$

$\therefore$ The distance covered is 72 km.

- (iv) $C = \frac{5}{3}d$

When $d = 0$, $C = 0$.

When $d = 3$, $C = 5$.



Practise Now 4

- (i) Total monthly cost of running the kindergarten

$$= \$5000 + 200 \times \$41$$

$$= \$13\,200$$

- (ii) Variable amount = \$20 580 – \$5000

$$= \$15\,580$$

$$\text{Number of children enrolled} = \frac{15\,580}{41} \\ = 380$$

- (iii) Variable amount = $n \times \$41$

$$= \$41n$$

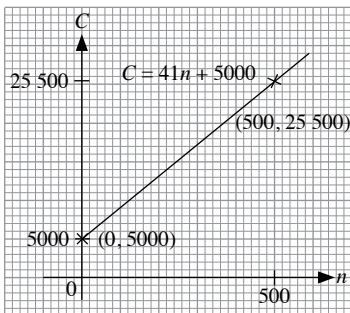
Total monthly cost = variable amount + fixed amount

$$\therefore C = 41n + 5000$$

- (iv) $C = 41n + 5000$

When $n = 0$, $C = 5000$.

When $n = 500$, $C = 25\,500$.



C is *not* directly proportional to n because the line does not pass through the origin.

Practise Now 5

- (a) Since $y = 6x^2$, i.e. $\frac{y}{x^2} = 6$ is a constant, then y is directly proportional to x^2 .

- (b) Since $\sqrt{y} = x^3$, i.e. $\frac{\sqrt{y}}{x^3} = 1$ is a constant, then $\sqrt{y}$ is directly proportional to x^3 .

Practise Now 6

1. (i) Since y is directly proportional to x^2 , then $y = kx^2$, where k is a constant.

When $x = 3$, $y = 18$,

$$18 = k \times 3^2$$

$$18 = 9k$$

$$\therefore k = 2$$

$$\therefore y = 2x^2$$

- (ii) When $x = 5$,

$$y = 2 \times 5^2$$

$$= 50$$

- (iii) When $y = 32$,

$$32 = 2x^2$$

$$x^2 = 16$$

$$\therefore x = \pm \sqrt{16}$$

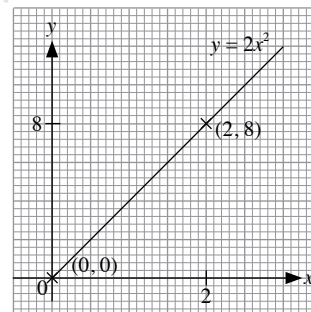
$$= \pm 4$$

- (iv) Since y is directly proportional to x^2 , then the graph of y against x^2 is a straight line that passes through the origin.

$$y = 2x^2$$

When $x = 0$, $y = 0$.

When $x = 2$, $y = 8$.



2. Since y is directly proportional to x^2 , then $y = kx^2$, where k is a constant.

When $x = 2$, $y = 21$,

$$21 = k \times 2^2$$

$$21 = 4k$$

$$\therefore k = \frac{21}{4}$$

$$\therefore y = \frac{21}{4}x^2$$

When $x = 4$,

$$y = \frac{21}{4} \times 4^2$$

$$= 84$$

3.	x	2	2.5	3	5	7
	y	36	56.25	81	225	441

Since y is directly proportional to x^2 ,

then $y = kx^2$, where k is a constant.

When $x = 3$, $y = 81$,

$$81 = k \times 3^2$$

$$\therefore k = 9$$

$$\therefore y = 9x^2$$

When $y = 56.25$,

$$56.25 = 9 \times x^2$$

$$x^2 = 6.25$$

$$x = \sqrt{6.25} \quad (x > 0)$$

$$= 2.5$$

When $y = 441$,

$$441 = 9 \times x^2$$

$$x^2 = 49$$

$$x = \sqrt{49} \quad (x > 0)$$

$$= 7$$

When $x = 2$,

$$y = 9 \times 2^2$$

$$= 36$$

When $x = 5$,

$$y = 9 \times 5^2$$

$$= 225$$

Practise Now 7

- (i) Since l is directly proportional to T^2 ,

then $l = kT^2$, where k is a constant.

When $T = 3$, $l = 220.5$,

$$220.5 = k \times 3^2$$

$$220.5 = 9k$$

$$\therefore k = 24.5$$

$$\therefore l = 24.5T^2$$

- (ii) When $T = 5$,

$$l = 24.5 \times 5^2$$

$$= 612.5$$

$\therefore$ The length of the pendulum is 612.5 cm.

- (iii) $0.98 \text{ m} = 98 \text{ cm}$

When $l = 98$,

$$98 = 24.5T^2$$

$$T^2 = 4$$

$$\therefore T = \sqrt{4} \quad (T > 0)$$

$$= 2$$

$\therefore$ The period of the pendulum is 2 s.

Practise Now 8

The time taken to fill the tank is inversely proportional to the number of taps used.

Method 1: Unitary Method

4 taps can fill the tank in 70 minutes.

1 tap can fill the tank in (70×4) minutes.

7 taps can fill the tank in $\frac{70 \times 4}{7} = 40$ minutes.

Method 2: Proportion Method

Let the time taken for 7 taps to fill the tank by y minutes.

Then $7y = 4 \times 70$ ($x_1y_1 = x_2y_2$)

$$y = \frac{4 \times 70}{7}$$

$$= 40$$

$\therefore$ 7 taps can fill the tank in 40 minutes.

Practise Now 9

- (a) The three variables are 'number of men', 'number of trenches' and 'number of hours'.

First, we keep the number of trenches constant.

Number of men	Number of trenches	Number of hours
3	2	5
1	2	5×3
5	2	$\frac{5 \times 3}{5} = 3$

Next, we keep the number of men constant.

Number of men	Number of trenches	Number of hours
5	2	3
5	1	$\frac{3}{2}$
5	7	$\frac{3}{2} \times 7 = 10.5$

$\therefore$ 5 men will take 10.5 hours to dig 7 trenches.

- (b) The three variables are 'number of taps', 'number of tanks' and 'number of minutes'.

First, we keep the number of tanks constant.

Number of taps	Number of tanks	Number of minutes
7	3	45
1	3	45×7
5	3	$\frac{45 \times 7}{5} = 6.3$

Next, we keep the number of taps constant.

Number of taps	Number of tanks	Number of minutes
5	3	63
5	1	$\frac{63}{3} = 21$

$\therefore$ 5 taps will take 21 minutes to fill one tank.

Practise Now 10

1. (i) When $x = 8$, (x increased by 4 times)

$$y = \frac{5}{4} \text{ (y decreased by 4 times)}$$

$$= 1.25$$

Alternatively,

$$x_2 y_2 = x_1 y_1$$

$$8 \times y = 2 \times 5$$

$$y = \frac{10}{8}$$

$$= 1.25$$

- (ii) Since y is inversely proportional to x ,

$$\text{then } y = \frac{k}{x}, \text{ where } k \text{ is a constant.}$$

When $x = 2$, $y = 5$,

$$5 = \frac{k}{2}$$

$$\therefore k = 10$$

$$\therefore y = \frac{10}{x}$$

- (iii) When $y = 10$,

$$10 = \frac{10}{x}$$

$$\therefore x = \frac{10}{10}$$

$$= 1$$

2. Since y is inversely proportional to x ,

$$x_2 y_2 = x_1 y_1$$

$$3 \times y = 2 \times 9$$

$$y = \frac{18}{3}$$

$$= 6$$

- 3.

x	0.5	1	2	3	5
y	8	4	2	$1\frac{1}{3}$	0.8

Since y is inversely proportional to x ,

$$\text{then } y = \frac{k}{x}, \text{ where } k \text{ is a constant.}$$

When $x = 2$, $y = 2$,

$$2 = \frac{k}{2}$$

$$\therefore k = 4$$

$$\therefore y = \frac{4}{x}$$

When $y = 4$,

$$4 = \frac{4}{x}$$

$$x = \frac{4}{4}$$

$$= 1$$

When $y = 0.8$,

$$0.8 = \frac{4}{x}$$

$$x = \frac{4}{0.8}$$

$$= 5$$

When $x = 0.5$,

$$y = \frac{4}{0.5}$$

$$= 8$$

When $x = 3$,

$$y = \frac{4}{3}$$

$$= 1\frac{1}{3}$$

Practise Now 11

- (i) Since I is inversely proportional to R ,

$$\text{then } I = \frac{k}{R}, \text{ where } k \text{ is a constant.}$$

When $R = 0.5$, $I = 12$,

$$12 = \frac{k}{0.5}$$

$$\therefore k = 6$$

$$\therefore I = \frac{6}{R}$$

When $R = 3$,

$$I = \frac{6}{3}$$

$$= 2$$

$\therefore$ The current flowing through the wire is 2 A.

- (ii) When $I = 3$,

$$3 = \frac{6}{R}$$

$$R = \frac{6}{3}$$

$$= 2$$

$\therefore$ The resistance of the wire is 2 Ω .

Practise Now 12

- (a) Since $y = \frac{4}{x^2}$, i.e. $x^2 y = 4$ is a constant, then y is inversely proportional to x^2 .

- (b) Since $y^2 = \frac{1}{\sqrt[3]{x}}$, i.e. $\sqrt[3]{x} y^2 = 1$ is a constant, then y^2 is inversely proportional to $\sqrt[3]{x}$.

- (c) Since $y = \frac{5}{x+2}$, i.e. $(x+2)y = 5$ is a constant, then y is inversely proportional to $x+2$.

Practise Now 13

1. (i) When $x = 8 = 2 \times 4$, (8 is 2 times of 4)

$$y = \frac{1}{2^2} \times 2 \quad (\text{y is } \frac{1}{2^2} \text{ times of 2 since y is inversely proportional to } x^2)$$

$$= \frac{1}{2}$$

- (ii) Since y is inversely proportional to x^2 ,

$$\text{then } y = \frac{k}{x^2}, \text{ where } k \text{ is a constant.}$$

$$\text{When } x = 4, y = 2,$$

$$2 = \frac{k}{4^2}$$

$$\therefore k = 32$$

$$\therefore y = \frac{32}{x^2}$$

- (iii) When $y = 8$,

$$8 = \frac{32}{x^2}$$

$$x^2 = \frac{32}{8}$$

$$= 4$$

$$x = \pm \sqrt{4}$$

$$= \pm 2$$

2. Since y is inversely proportional to $\sqrt{x}$,

$$\text{then } y = \frac{k}{\sqrt{x}}, \text{ where } k \text{ is a constant.}$$

$$\text{When } x = 9, y = 6,$$

$$6 = \frac{k}{\sqrt{9}}$$

$$\therefore k = 18$$

$$\therefore y = \frac{18}{\sqrt{x}}$$

$$\text{When } x = 25,$$

$$y = \frac{18}{\sqrt{25}}$$

$$= 3.6$$

3.

x	0.25	1	4	16	36
y	16	8	2	2	$1\frac{1}{3}$

Since y is inversely proportional to $\sqrt{x}$,

$$\text{then } y = \frac{k}{\sqrt{x}}, \text{ where } k \text{ is a constant.}$$

$$\text{When } x = 1, y = 8,$$

$$8 = \frac{k}{\sqrt{1}}$$

$$\therefore k = 8$$

$$\therefore y = \frac{8}{\sqrt{x}}$$

When $y = 16$,

$$16 = \frac{8}{\sqrt{x}}$$

$$\sqrt{x} = \frac{8}{16}$$

$$= \frac{1}{2}$$

$$x = \left(\frac{1}{2}\right)^2$$

$$= 0.25$$

When $y = 1\frac{1}{3}$,

$$1\frac{1}{3} = \frac{8}{\sqrt{x}}$$

$$\sqrt{x} = \frac{8}{1\frac{1}{3}}$$

$$= 6$$

$$x = 6^2$$

$$= 36$$

When $x = 4$,

$$y = \frac{8}{\sqrt{4}}$$

$$= 4$$

When $x = 16$,

$$y = \frac{8}{\sqrt{16}}$$

$$= 2$$

Practise Now 14

- (i) Since F is inversely proportional to d ,

$$\text{then } F = \frac{k}{d^2}, \text{ where } k \text{ is a constant.}$$

When $d = 2, F = 10$,

$$10 = \frac{k}{2^2}$$

$$10 = \frac{k}{4}$$

$$\therefore k = 40$$

$$\therefore F = \frac{40}{d^2}$$

When $d = 5$,

$$F = \frac{40}{5^2}$$

$$= 1.6$$

$\therefore$ The force between the particles is 1.6 N.

(ii) When $F = 25$,

$$25 = \frac{40}{d^2}$$

$$d^2 = \frac{40}{25}$$

$$= \frac{8}{5}$$

$$d = \sqrt{\frac{8}{5}} \quad (d > 0)$$

$$= 1.26 \text{ (to 3 s.f.)}$$

$\therefore$ The distance between the particles is 1.26 m.

Exercise 1A

1. (i) The number of books is directly proportional to the mass of books.

108 books have a mass of 30 kg.

1 book has a mass of $\frac{30}{108}$ kg.

150 books have a mass of $\frac{30}{108} \times 150 = 41 \frac{2}{3}$ kg.

- (ii) The mass of books is directly proportional to the number of books.

30 kg is the mass of 108 books.

1 kg is the mass of $\frac{108}{30}$ books.

20 kg is the mass of $\frac{108}{30} \times 20 = 72$ books.

2. (i) The number of books is directly proportional to the length occupied by the books.

60 books occupy a length of 1.5 m.

1 book occupies a length of $\frac{1.5}{60}$ m.

50 books occupy a length of $\frac{1.5}{60} \times 50 = 1.25$ m.

- (ii) The length occupied by the books is directly proportional to the number of books.

$$1.5 \text{ m} = (1.5 \times 100) \text{ cm}$$

$$= 150 \text{ cm}$$

150 cm is the length occupied by 60 books.

1 cm is the length occupied by $\frac{60}{150}$ books.

80 cm is the length occupied by $\frac{60}{150} \times 80 = 32$ books.

3. (i) Since y is directly proportional to x ,
then $y = kx$, where k is a constant.

When $x = 4.5$, $y = 3$,

$$3 = k \times 4.5$$

$$\therefore k = \frac{2}{3}$$

$$\therefore y = \frac{2}{3}x$$

(ii) When $y = 6$,

$$6 = \frac{2}{3}x$$

$$x = 6 \times \frac{3}{2}$$

$$= 9$$

(iii) When $x = 12$,

$$y = \frac{2}{3} \times 12$$

$$= 8$$

4. (i) Since Q is directly proportional to P ,
then $Q = kP$, where k is a constant.

When $P = 4$, $Q = 28$,

$$28 = k \times 4$$

$$\therefore k = 7$$

$$\therefore Q = 7P$$

(ii) When $P = 5$,

$$Q = 7 \times 5$$

$$= 35$$

(iii) When $Q = 42$,

$$42 = 7 \times P$$

$$P = 6$$

5. (a) The mass of tea leaves is directly proportional to the cost of tea leaves.

3 kg of tea leaves cost \$18.

1 kg of tea leaves cost $\left(\frac{18}{3}\right)$.

10 kg of tea leaves cost $\left(\frac{18}{3} \times 10\right) = \$60$.

- (b) The mass of sugar is directly proportional to the cost.

b kg of sugar cost \$ c .

1 kg of sugar cost $\$ \frac{c}{b}$.

a kg of sugar cost $\left(\frac{c}{b} \times a\right) = \$ \frac{ac}{b}$.

6. The amount of metal is directly proportional to the mass of the metal.

$\frac{5}{9}$ of a piece of metal has a mass of 7 kg.

A whole piece of metal has a mass of $\frac{7}{\frac{5}{9}}$ kg.

$\frac{2}{7}$ of a piece of metal has a mass of $\frac{7}{\frac{5}{9}} \times \frac{2}{7} = 3 \frac{3}{5}$ kg.

7. Since z is directly proportional to x ,

$$\frac{x_2}{z_2} = \frac{x_1}{z_1}$$

$$\frac{x}{18} = \frac{3}{12}$$

$$x = \frac{3}{12} \times 18$$

$$= 4.5$$

8. Since B is directly proportional to A ,

$$\frac{B_2}{A_2} = \frac{B_1}{A_1}$$

$$\frac{B}{24} = \frac{3}{18}$$

$$B = \frac{3}{18} \times 24 \\ = 4$$

9. (a)

x	4	20	24	36	44
y	1	5	6	9	11

Since y is directly proportional to x , then $y = kx$, where k is a constant.

When $x = 24$, $y = 6$,

$$6 = k \times 24$$

$$\therefore k = \frac{1}{4}$$

$$\therefore y = \frac{1}{4}x$$

When $y = 9$,

$$9 = \frac{1}{4} \times x$$

$$x = 9 \times 4 \\ = 36$$

When $y = 11$,

$$11 = \frac{1}{4} \times x$$

$$x = 11 \times 4 \\ = 44$$

When $x = 4$,

$$y = \frac{1}{4} \times 4 \\ = 1$$

When $x = 20$,

$$y = \frac{1}{4} \times 20 \\ = 5$$

(b)

x	2	3	5.5	8	9.5
y	2.4	3.6	6.6	9.6	11.4

Since y is directly proportional to x , then $y = kx$, where k is a constant.

When $x = 3$, $y = 3.6$,

$$3.6 = k \times 3$$

$$\therefore k = 1.2$$

$$\therefore y = 1.2x$$

When $y = 9.6$,

$$9.6 = 1.2 \times x$$

$$x = \frac{9.6}{1.2} \\ = 8$$

When $y = 11.4$,

$$11.4 = 1.2 \times x$$

$$x = \frac{11.4}{1.2} \\ = 9.5$$

When $x = 2$,

$$y = 1.2 \times 2 \\ = 2.4$$

When $x = 5.5$,

$$y = 1.2 \times 5.5 \\ = 6.6$$

10. (i) Since y is directly proportional to x , then $y = kx$, where k is a constant.

When $x = 5$, $y = 20$,

$$20 = k \times 5$$

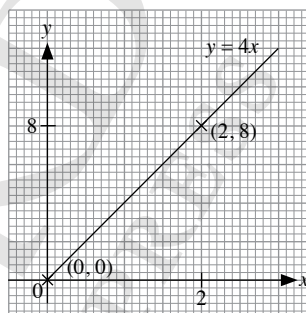
$$\therefore k = 4$$

$$\therefore y = 4x$$

- (ii) $y = 4x$

When $x = 0$, $y = 0$.

When $x = 2$, $y = 8$.



11. (i) Since z is directly proportional to y , then $z = ky$, where k is a constant.

When $y = 6$, $z = 48$,

$$48 = k \times 6$$

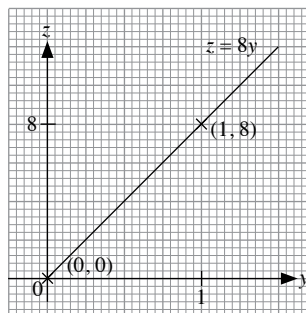
$$\therefore k = 8$$

$$\therefore z = 8y$$

- (ii) $z = 8y$

When $y = 0$, $z = 0$.

When $x = 1$, $y = 8$.



12. (i) Since F is directly proportional to m , then $F = km$, where k is a constant.

When $m = 5$, $F = 49$,

$$49 = k \times 5$$

$$\therefore k = 9.8$$

$$\therefore F = 9.8m$$

- (ii) When $m = 14$,

$$F = 9.8 \times 14$$

$$= 137.2$$

(iii) When $F = 215.6$,

$$215.6 = 9.8 \times m$$

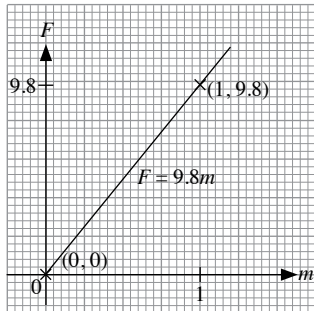
$$m = \frac{215.6}{9.8}$$

$$= 22$$

(iv) $F = 9.8m$

When $m = 0$, $F = 0$.

When $m = 1$, $F = 9.8$.



13. (i) Since P is directly proportional to T ,
then $P = kT$, where k is a constant.

When $T = 10$, $y = 25$,

$$25 = k \times 10$$

$$\therefore k = 2.5$$

$$\therefore P = 2.5T$$

(ii) When $T = 24$,

$$P = 2.5 \times 24$$

$$= 60$$

(iii) When $P = 12$,

$$12 = 2.5 \times T$$

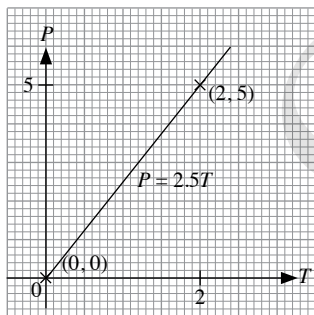
$$T = \frac{12}{2.5}$$

$$= 4.8$$

(iv) $P = 2.5T$

When $T = 0$, $P = 0$.

When $T = 2$, $P = 5$.



14. (i) Since V is directly proportional to R ,
then $V = kR$, where k is a constant.

When $R = 6$, $V = 9$,

$$9 = k \times 6$$

$$\therefore k = 1.5$$

$$\therefore V = 1.5R$$

(ii) When $R = 15$,

$$V = 1.5 \times 15$$

$$= 22.5$$

(iii) When $V = 15$,

$$15 = 1.5 \times R$$

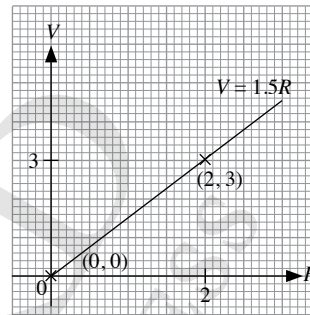
$$R = \frac{15}{1.5}$$

$$= 10$$

(iv) $V = 1.5R$

When $R = 0$, $V = 0$.

When $R = 2$, $V = 3$.



15. (i) Total income for that month

$$= \$600 + \$8 \times 95$$

$$= \$1360$$

(ii) Variable amount = $\$1680 - \$600$

$$= \$1080$$

$$\text{Number of tyres he sells in that month} = \frac{1080}{8}$$

$$= 135$$

(iii) Variable amount = $n \times \$8$

$$= \$8n$$

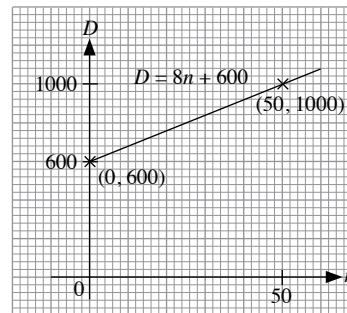
Total income = variable amount + fixed amount

$$\therefore D = 8n + 600$$

(iv) $D = 8n + 600$

When $n = 0$, $D = 600$.

When $n = 50$, $D = 1000$.



D is not directly proportional to n because the line does not pass through the origin.

16. Let the mass of ice produced be m tonnes,
the number of hours of production be T hours.
Since m is directly proportional to T ,
then $m = kT$, where k is a constant.

$$\text{When } T = \frac{30}{60} - \frac{10}{60} = \frac{1}{3}, m = 20,$$

$$20 = k \times \frac{1}{3}$$

$$\therefore k = 60$$

$$\therefore m = 60T$$

$$\text{When } T = 1.75 - \frac{10}{60},$$

$$m = 60 \left(1.75 - \frac{1}{6} \right)$$

$$= 95$$

$\therefore$ The mass of ice manufactured is 95 tonnes.

Exercise 1B

1. (i) Since x is directly proportional to y^3 ,
then $x = ky^3$, where k is a constant.

$$\text{When } y = 2, x = 32,$$

$$32 = k \times 2^3$$

$$32 = 8k$$

$$\therefore k = 4$$

$$\therefore x = 4y^3$$

- (ii) When $y = 6$,

$$x = 4 \times 6^3$$

$$= 864$$

- (iii) When $x = 108$,

$$108 = 4 \times y^3$$

$$y^3 = \frac{108}{4}$$

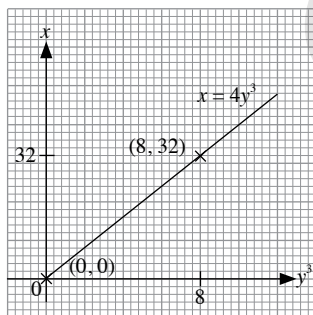
$$= 27$$

$$y = 3$$

- (iv) $x = 4y^3$

$$\text{When } y = 0, x = 0.$$

$$\text{When } y = 2, x = 32.$$



2. (i) Since z^2 is directly proportional to w ,
then $z^2 = kw$, where k is a constant.

$$\text{When } w = 8, z = 4,$$

$$4^2 = k \times 8$$

$$16 = 8k$$

$$\therefore k = 2$$

$$\therefore z^2 = 2w$$

- (ii) When $w = 18$,

$$z^2 = 2 \times 18$$

$$= 36$$

$$z = \pm \sqrt{36}$$

$$= \pm 6$$

- (iii) When $z = 5$,

$$5^2 = 2 \times w$$

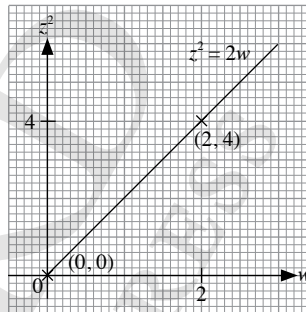
$$w = \frac{25}{2}$$

$$= 12.5$$

- (iv) $z^2 = 2w$

$$\text{When } w = 0, z^2 = 0.$$

$$\text{When } w = 2, z^2 = 4.$$



3. (i) Since y is directly proportional to x^n ,
then $y = kx^n$, where k is a constant.
Since $y \text{ m}^2$ is the area of a square of length $x \text{ m}$, then $y = x^2$.
 $kx^n = x^2$
 $\therefore n = 2$
- (ii) Since y is directly proportional to x^n ,
then $y = kx^n$, where k is a constant.
Since $y \text{ cm}^3$ is the volume of a cube of length $x \text{ cm}$, then $y = x^3$.
 $kx^n = x^3$
 $\therefore n = 3$
4. (a) Since $y = 4x^2$, i.e. $\frac{y}{x^2} = 4$ is a constant, then y is directly proportional to x^2 .
- (b) Since $y = 3\sqrt{x}$, i.e. $\frac{y}{\sqrt{x}} = 3$ is a constant, then y is directly proportional to $\sqrt{x}$.
- (c) Since $y^2 = 5x^3$, i.e. $\frac{y^2}{x^3} = 5$ is a constant, then y^2 is directly proportional to x^3 .
- (d) Since $p^3 = q^2$, i.e. $\frac{p^3}{q^2} = 1$ is a constant, then p^3 is directly proportional to q^2 .

5. Since z^2 is directly proportional to x^3 ,

$$\begin{aligned}\frac{z_2^2}{x_2^3} &= \frac{z_1^2}{x_1^3} \\ \frac{z^2}{9^3} &= \frac{8^2}{4^3} \\ z^2 &= \frac{8^2}{4^3} \times 9^3 \\ &= 729 \\ z &= \pm \sqrt{729} \\ &= \pm 27\end{aligned}$$

6. Since q is directly proportional to $(p-1)^2$,

$$\begin{aligned}\frac{(p_2-1)^2}{q_2} &= \frac{(p_1-1)^2}{q_1} \\ \frac{(p-1)^2}{80} &= \frac{(3-1)^2}{20} \\ (p-1)^2 &= \frac{(3-1)^2}{20} \times 80 \\ &= 16\end{aligned}$$

$$\begin{aligned}p-1 &= -4 \quad \text{or} \quad p-1 = 4 \\ p &= -3 \quad \quad \quad p = 5 \\ \therefore p &= -3 \text{ or } 5\end{aligned}$$

7.

x	3	4	5	6	7
y	81	192	375	648	1029

Since y is directly proportional to x^3 ,
then $y = kx^3$, where k is a constant.

When $x = 6$, $y = 648$,

$$648 = k \times 6^3$$

$$\therefore k = 3$$

$$\therefore y = 3x^3$$

When $y = 375$,

$$375 = 3 \times x^3$$

$$x^3 = 125$$

$$x = \sqrt[3]{125}$$

$$= 5$$

When $y = 1029$,

$$1029 = 3 \times x^3$$

$$x^3 = 343$$

$$x = \sqrt[3]{343}$$

$$= 7$$

When $x = 3$,

$$y = 3 \times 3^3$$

$$= 81$$

When $x = 4$,

$$y = 3 \times 4^3$$

$$= 192$$

8.

r	0.2	0.5	0.7	1.5	1.8
m	0.016	0.25	0.686	6.75	11.664

Since m is directly proportional to r^3 ,

then $m = kr^3$, where k is a constant.

When $r = 1.5$, $m = 6.75$,

$$6.75 = k \times 1.5^3$$

$$\therefore k = 2$$

$$\therefore m = 2r^3$$

When $m = 0.25$,

$$0.25 = 2 \times r^3$$

$$r^3 = 0.125$$

$$r = \sqrt[3]{0.125}$$

$$= 0.5$$

When $m = 11.664$,

$$11.664 = 2 \times r^3$$

$$r^3 = 5.832$$

$$r = \sqrt[3]{5.832}$$

$$= 1.8$$

When $r = 0.2$,

$$m = 2 \times 0.2^3$$

$$= 0.016$$

When $r = 0.7$,

$$m = 2 \times 0.7^3$$

$$= 0.686$$

9. (i) Since L is directly proportional to $\sqrt{N}$,

then $L = k\sqrt{N}$, where k is a constant.

When $N = 1$, $L = 2.5$,

$$2.5 = k\sqrt{1}$$

$$\therefore k = 2.5$$

$$\therefore L = 2.5\sqrt{N}$$

- (ii) When $N = 4$,

$$L = 2.5 \times \sqrt{4}$$

$$= 5$$

$\therefore$ The length 4 hours after its birth is 5 cm.

- (iii) When $L = 15$,

$$15 = 2.5 \times \sqrt{N}$$

$$\sqrt{N} = 6$$

$$N = 6^2$$

$$= 36$$

$\therefore$ It will take 36 hours for the earthworm to grow to a length of 15 cm.

10. Since y is directly proportional to x^2 ,

then $y = kx^2$, where k is a constant.

When $x = 1$,

$$y = k \times 1^2$$

$$= k$$

When $x = 3$,

$$y = k \times 3^2$$

$$= 9k$$

Since the difference in the values of y is 32,

$$9k - k = 32$$

$$8k = 32$$

$$\therefore k = 4$$

$$\therefore y = 4x^2$$

When $x = -2$,

$$y = 4 \times (-2)^2$$

$$= 16$$

11. Since y is directly proportional to x^2 ,

$$\frac{y_2}{x_2^2} = \frac{y_1}{x_1^2}$$

$$\frac{y}{(2x)^2} = \frac{a}{x^2}$$

$$y = \frac{a}{x^2} \times (2x)^2$$

$$= \frac{a}{x^2} \times 4x^2$$

$$= 4a$$

12. Let the braking distance of a vehicle be D m,
the speed of the vehicle be B m/s.

Since D is directly proportional to B^2 , then $D = kB^2$, where k is a constant.

When $B = b$, $D = d$,

$$d = k \times b^2$$

$$\therefore k = \frac{d}{b^2}$$

$$\therefore D = \frac{d}{b^2} B^2$$

When the speed of the vehicle is increased by 200%,

$$B = (100\% + 200\%) \times b$$

$$= \frac{100 + 200}{100} \times b$$

$$= 3b$$

When $B = 3b$,

$$D = \frac{d}{b^2} (3b)^2$$

$$= \frac{d}{b^2} (9b^2)$$

$$= 9d$$

Percentage increase in its braking distance

$$= \frac{9d - d}{d} \times 100\%$$

$$= 800\%$$

Exercise 1C

1. (a) The number of pencils is directly proportional to the total cost of the pencils.

Assumption: All pencils are identical and cost the same each.

- (b) The number of taps filling a tank is inversely proportional to the time taken to fill the tank.

Assumption: All taps are identical and each tap takes the same time to fill the tank.

- (c) The number of men laying a road is inversely proportional to the taken to finish laying the road.

Assumption: All the men work at the same rate in laying the road.

- (d) The number of cattle to be fed is directly proportional to the amount of fodder.

Assumption: All the cattle eat the same amount of fodder.

- (e) The number of cattle to be fed is inversely proportional to the time taken to finish a certain amount of the fodder.

Assumption: All the cattle eat the fodder at the same rate.

$\therefore$ (b), (c) and (e) are in inverse proportion.

2. The number of men to build a bridge is inversely proportional to the number of days to build the bridge.

8 men can build a bridge in 12 days.

1 man can build the bridge in (12×8) days.

6 men can build the bridge in $\frac{12 \times 8}{6} = 16$ days.

The assumption made is that all the men work at the same rate in building the bridge.

3. (i) Since x is inversely proportional to y ,

$$y_2 x_2 = y_1 x_1$$

$$25 \times x = 5 \times 40$$

$$x = \frac{5 \times 40}{25}$$

$$= 8$$

- (ii) Since x is inversely proportional to y ,

then $x = \frac{k}{y}$, where k is a constant.

When $y = 5$, $x = 40$,

$$40 = \frac{k}{5}$$

$$\therefore k = 200$$

$$\therefore x = \frac{200}{y}$$

- (iii) When $x = 400$,

$$400 = \frac{200}{y}$$

$$y = \frac{200}{400}$$

$$= 0.5$$

4. (i) Since Q is inversely proportional to P ,

then $Q = \frac{k}{P}$, where k is a constant.

When $P = 2$, $Q = 0.25$,

$$0.25 = \frac{k}{2}$$

$$\therefore k = 0.5$$

$$\therefore Q = \frac{0.5}{P}$$

$$= \frac{1}{2P}$$

- (ii) When $P = 5$,

$$Q = \frac{1}{2(5)}$$

$$= 0.1$$

- (iii) When $Q = 0.2$,

$$0.2 = \frac{1}{2P}$$

$$2P = \frac{1}{0.2}$$

$$= 5$$

$$P = 2.5$$

5. The number of days is inversely proportional to the number of workers employed.

16 days are needed for 35 workers to complete the project.

1 day is needed for (35×16) workers to complete the project.

14 days are needed for $\frac{35 \times 16}{14} = 40$ workers to complete the project.

Number of additional workers to employ = $40 - 35$
 $= 5$

6. (i) The number of days is inversely proportional to the number of cattle to consume a consignment of fodder.

50 days are needed for 1260 cattle to consume a consignment of fodder.

1 day is needed for (1260×50) cattle to consume a consignment of fodder.

75 days are needed for $\frac{1260 \times 50}{75} = 840$ cattle to consume a consignment of fodder.

- (ii) 1260 cattle consume a consignment of fodder in 50 days.

1 cattle consume a consignment of fodder in (50×1260) days.

1575 cattle consume a consignment of fodder in

$$\frac{50 \times 1260}{1575} = 40 \text{ days.}$$

7. The number of athletes is inversely proportional to the number of days the food can last.

72 athletes take 6 days to consume the food.

1 athlete takes (6×72) days to consume the food.

$72 - 18 = 54$ athletes take $\frac{6 \times 72}{54} = 8$ days to consume the food.

Number of additional days the food can last = $8 - 6$
 $= 2$ days

The assumption made is that all athletes consume the same amount of food every day.

8. Since z is inversely proportional to x ,

$$x_2 z_2 = x_1 z_1$$

$$x \times 70 = 7 \times 5$$

$$x = \frac{7 \times 5}{70}$$

$$= 0.5$$

9. Since B is inversely proportional to A ,

$$A_2 B_2 = A_1 B_1$$

$$1.4 \times B = 2 \times 3.5$$

$$B = \frac{2 \times 3.5}{1.4}$$

$$= 5$$

10. (a)

x	0.5	2	2.5	3	8
y	24	6	4.8	4	1.5

Since y is inversely proportional to x ,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 3$, $y = 4$,

$$4 = \frac{k}{3}$$

$$\therefore k = 12$$

$$\therefore y = \frac{12}{x}$$

When $y = 24$,

$$24 = \frac{12}{x}$$

$$x = \frac{12}{24}$$

$$= 0.5$$

When $y = 1.5$,

$$1.5 = \frac{12}{x}$$

$$x = \frac{12}{1.5}$$

$$= 8$$

When $x = 2$,

$$y = \frac{12}{2}$$

$$= 6$$

When $x = 2.5$,

$$y = \frac{12}{2.5}$$

$$= 4.8$$

(b)	x	3	4	4.5	14.4	25
	y	12	9	8	2.5	1.44

Since y is inversely proportional to x ,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 4$, $y = 9$,

$$9 = \frac{k}{4}$$

$$\therefore k = 36$$

$$\therefore y = \frac{36}{x}$$

When $y = 8$,

$$8 = \frac{36}{x}$$

$$x = \frac{36}{8}$$

$$= 4.5$$

When $y = 2.5$,

$$2.5 = \frac{36}{x}$$

$$x = \frac{36}{2.5}$$

$$= 14.4$$

When $x = 3$,

$$y = \frac{36}{3}$$

$$= 12$$

When $x = 25$,

$$y = \frac{36}{25}$$

$$= 1.44$$

11. (i) Since f is inversely proportional to λ ,

then $f = \frac{k}{\lambda}$, where k is a constant.

When $\lambda = 3000$, $f = 100$,

$$100 = \frac{k}{3000}$$

$$\therefore k = 300\,000$$

$$\therefore f = \frac{300\,000}{\lambda}$$

When $\lambda = 500$,

$$f = \frac{300\,000}{500}$$

$$= 600$$

$\therefore$ The frequency of the radio wave is 600 kHz.

- (ii) When $f = 800$,

$$800 = \frac{300\,000}{\lambda}$$

$$\lambda = \frac{300\,000}{800}$$

$$= 375$$

$\therefore$ The wavelength of the radio wave is 375 m.

12. (i) Since t is inversely proportional to N ,

then $t = \frac{k}{N}$, where k is a constant.

When $N = 3$, $t = 8$,

$$8 = \frac{k}{3}$$

$$\therefore k = 24$$

$$\therefore t = \frac{24}{N}$$

- (ii) When $N = 6$,

$$t = \frac{24}{6}$$

$$= 4$$

$\therefore$ The number of hours needed by 6 men is 4 hours.

- (iii) When $t = \frac{3}{4}$,

$$\frac{3}{4} = \frac{24}{N}$$

$$N = 24 \times \frac{4}{3}$$

$$= 32$$

$\therefore$ 32 men need to be employed.

13. The three variables are 'number of glassblowers', 'number of vases' and 'number of minutes'.

First, we keep the number of vases constant.

Number of glassblowers	Number of vases	Number of minutes
12	12	9
1	12	9×12
8	12	$\frac{9 \times 12}{8} = 13.5$

Next, we keep the number of glassblowers constant.

Number of glassblowers	Number of vases	Number of minutes
8	12	13.5
8	1	$\frac{13.5}{12}$
8	32	$\frac{13.5}{12} \times 32 = 36$

$\therefore$ 8 glassblowers will take 36 minutes to make 32 vases.

14. The three variables are 'number of sheep', 'number of consignments' and 'number of days'.

First, we keep the number of consignments constant.

Number of sheep	Number of consignments	Number of days
100	1	20
1	1	20×1000
550	1	$\frac{20 \times 1000}{550} = 36 \frac{4}{11}$

Next, we keep the number of sheep constant.

Number of sheep	Number of consignments	Number of days
550	1	$36 \frac{4}{11}$
550	$400 \div 36 \frac{4}{11} = 11$	400

$\therefore$ 11 consignments of fodder are needed.

15. In 1 minute, tap A alone fills up $\frac{1}{6}$ of the tank.

In 1 minute, tap B alone fills up $\frac{1}{9}$ of the tank.

In 1 minute, pipe C alone empties $\frac{1}{15}$ of the tank.

In 1 minute, when both taps and the pipe are turned on,

$$\frac{1}{6} + \frac{1}{9} - \frac{1}{15} = \frac{19}{90} \text{ of the tank is filled up.}$$

$$\text{Time to fill up the tank} = \frac{90}{19}$$

$$= 4 \frac{14}{19} \text{ minutes}$$

16. Total number of hours worked on the road after 20 working days

$$= 20 \times 50 \times 8$$

$$= 8000 \text{ hours}$$

The length of the road laid is directly proportional to the number of hours.

1200 m of road is laid in 8000 hours.

$$1 \text{ m of road is laid in } \frac{8000}{1200} \text{ hours.}$$

$$3000 - 1200 = 1800 \text{ m of road is laid in } \frac{8000}{1200} \times 1800 = 12000 \text{ hours.}$$

Let the number of additional men to employ be x .

$$(30 - 20) \times (50 + x) \times 10 = 12000$$

$$100(50 + x) = 12000$$

$$50 + x = 120$$

$$x = 70$$

$\therefore$ 70 more men needs to be employed.

Exercise 1D

1. (i) Since x is inversely proportional to y^3 ,

$$y_2^3 x_2 = y_1^3 x_1$$

$$4^3 \times x = 2^3 \times 50$$

$$x = \frac{2^3 \times 50}{4^3}$$

$$= 6.25$$

(ii) Since x is inversely proportional to y^3 ,

$$\text{then } x = \frac{k}{y^3}, \text{ where } k \text{ is a constant.}$$

$$\text{When } y = 2, x = 50,$$

$$50 = \frac{k}{2^3}$$

$$\therefore k = 400$$

$$\therefore x = \frac{400}{y^3}$$

(iii) When $x = 3.2$,

$$3.2 = \frac{400}{y^3}$$

$$y^3 = \frac{400}{3.2}$$

$$= 125$$

$$y = \sqrt[3]{125}$$

$$= 5$$

2. (i) Since z is inversely proportional to $\sqrt{w}$,

$$\text{then } z = \frac{k}{\sqrt{w}}, \text{ where } k \text{ is a constant.}$$

$$\text{When } w = 9, z = 9,$$

$$9 = \frac{k}{\sqrt{9}}$$

$$\therefore k = 27$$

$$\therefore z = \frac{27}{\sqrt{w}}$$

(ii) When $w = 16$,

$$z = \frac{27}{\sqrt{16}}$$

$$= 6.75$$

(iii) When $z = 3$,

$$3 = \frac{27}{\sqrt{w}}$$

$$\sqrt{w} = \frac{27}{3}$$

$$= 9$$

$$w = 9^2$$

$$= 81$$

3. (a) Since $y = \frac{3}{x^2}$, i.e. $yx^2 = 3$ is a constant, then y is inversely proportional to x^2 .

(b) Since $y = \frac{1}{\sqrt{x}}$, i.e. $y\sqrt{x} = 1$ is a constant, then y is inversely proportional to $\sqrt{x}$.

(c) Since $y^2 = \frac{5}{x^3}$, i.e. $y^2 x^3 = 5$ is a constant, then y^2 is inversely proportional to x^3 .

(d) Since $n = \frac{7}{m-1}$, i.e. $n(m-1) = 7$ is a constant, then n is inversely proportional to $m-1$.

(e) Since $q = \frac{4}{(p+1)^2}$, i.e. $q(p+1)^2 = 4$ is a constant, then q is inversely proportional to $(p+1)^2$.

4. Since z is inversely proportional to $\sqrt[3]{x}$,

$$\sqrt[3]{x_2 z_2} = \sqrt[3]{x_1 z_1}$$

$$\sqrt[3]{216} \times z = \sqrt[3]{64} \times 5$$

$$z = \frac{\sqrt[3]{64} \times 5}{\sqrt[3]{216}}$$

$$= 3 \frac{1}{3}$$

5. Since q^2 is inversely proportional to $p + 3$,

$$\begin{aligned}(p_2 + 3)q_2^2 &= (p_1 + 3)q_1^2 \\ (17 + 3) \times q^2 &= (2 + 3) \times 5^2 \\ 20q^2 &= 125 \\ q^2 &= \frac{125}{20} \\ &= 6.25 \\ q &= \pm \sqrt{6.25} \\ &= \pm 2.5\end{aligned}$$

6.

s	1	2	4	10	20
t	80	10	1.25	0.08	0.01

Since t is inversely proportional to s^3 ,

then $t = \frac{k}{s^3}$, where k is a constant.

When $s = 1$, $t = 80$,

$$80 = \frac{k}{1^3}$$

$$\therefore k = 80$$

$$\therefore t = \frac{80}{s^3}$$

When $t = 0.08$,

$$0.08 = \frac{80}{s^3}$$

$$\begin{aligned}s^3 &= \frac{80}{0.08} \\ &= 1000\end{aligned}$$

$$\begin{aligned}s &= \sqrt[3]{1000} \\ &= 10\end{aligned}$$

When $t = 0.01$,

$$0.01 = \frac{80}{s^3}$$

$$\begin{aligned}s^3 &= \frac{80}{0.01} \\ &= 8000\end{aligned}$$

$$\begin{aligned}s &= \sqrt[3]{8000} \\ &= 20\end{aligned}$$

When $s = 2$,

$$t = \frac{80}{2^3}$$

$$= 10$$

When $s = 4$,

$$t = \frac{80}{4^3}$$

$$= 1.25$$

7. (i) Since F is inversely proportional to d^2 ,

then $F = \frac{k}{d^2}$, where k is a constant.

- (ii) When $F = 20$, let the distance between the particles be x m.

$$20 = \frac{k}{x^2}$$

$$\therefore k = 20x^2$$

$$\therefore F = \frac{20x^2}{d^2}$$

When the distance is halved, i.e. $d = \frac{1}{2}x$,

$$F = \frac{20x^2}{\left(\frac{1}{2}x\right)^2}$$

$$= 20x^2 \times \frac{4}{x^2}$$

$$= 80$$

$\therefore$ The force is 80 N.

8. (i) For a fixed volume, since h is inversely proportional to r^2 , then

$h = \frac{k}{r^2}$, where k is a constant.

When $r = 6$, $h = 5$,

$$5 = \frac{k}{6^2}$$

$$\therefore k = 180$$

$$\therefore h = \frac{180}{r^2}$$

When $r = 3$,

$$h = \frac{180}{3^2}$$

$$= 20$$

$\therefore$ The height of cone B is 20 cm.

- (ii) When $h = 1.25$,

$$1.25 = \frac{180}{r^2}$$

$$r^2 = \frac{180}{1.25}$$

$$= 144$$

$$r = \sqrt{144} \quad (r > 0)$$

$$= 12$$

$\therefore$ The base radius of cone C is 12 cm.

9. Since y is inversely proportional to $2x + 1$,

then $y = \frac{k}{2x + 1}$, where k is a constant.

When $x = 0.5$,

$$y = \frac{k}{2(0.5) + 1}$$

$$= \frac{k}{2}$$

When $x = 2$,

$$y = \frac{k}{2(2) + 1}$$

$$= \frac{k}{5}$$

Since the difference in the values of y is 0.9,

$$\frac{k}{2} - \frac{k}{5} = 0.9$$

$$0.3k = 0.9$$

$$\therefore k = 3$$

$$\therefore y = \frac{3}{2x+1}$$

When $x = -0.25$,

$$y = \frac{3}{2(-0.25)+1} = 6$$

10. Since y is inversely proportional to x^2 ,

$$x_2^2 y_2 = x_1^2 y_1$$

$$(3x)^2 y = x^2 b$$

$$9x^2 y = bx^2$$

$$y = \frac{bx^2}{9x^2}$$

$$= \frac{1}{9} b$$

11. Let the force of attraction between two magnets be X N,
the distance between two magnets be Y cm.

Since X is inversely proportional to Y^2 ,

then $X = \frac{k}{Y^2}$, where k is a constant.

When $X = F$, $Y = r$,

$$F = \frac{k}{r^2}$$

$$\therefore k = Fr^2$$

$$\therefore X = \frac{Fr^2}{Y^2}$$

When the distance between the magnets is increased by 400%,

$$Y = (100\% + 400\%) \times r$$

$$= \frac{100 + 400}{100} \times r$$

$$= 5r$$

When $Y = 5r$,

$$X = \frac{Fr^2}{(5r)^2}$$

$$= \frac{Fr^2}{25r^2}$$

$$= 0.04F$$

Comparing with $X = cF$,

$\therefore$ The value of c is 0.04.

Review Exercise 1

1. (i) Since y is directly proportional to x ,
then $y = kx$, where k is a constant.
When $x = 2$, $y = 6$,
 $6 = k \times 2$
 $\therefore k = 3$
 $\therefore y = 3x$

- (ii) When $x = 11$,

$$y = 3 \times 11 = 33$$

- (iii) When $y = 12$,

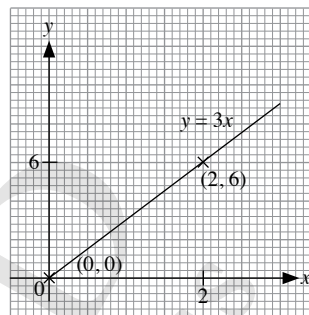
$$12 = 3 \times x$$

$$x = 4$$

- (iv) $y = 3x$

When $x = 0$, $y = 0$.

When $x = 2$, $y = 6$.



2. (i) Since A is directly proportional to B ,
then $A = kB$, where k is a constant.

$$\text{When } B = \frac{5}{6}, A = 1\frac{2}{3},$$

$$1\frac{2}{3} = k \times \frac{5}{6}$$

$$\therefore k = 2$$

$$\therefore A = 2B$$

$$\text{When } B = \frac{1}{3},$$

$$A = 2 \times \frac{1}{3}$$

$$= \frac{2}{3}$$

- (ii) When $A = 7\frac{1}{2}$,

$$7\frac{1}{2} = 2 \times B$$

$$B = 3\frac{3}{4}$$

3. (i) Since y is directly proportional to x^3 , then $y = kx^3$,
where k is a constant.

When $x = 3$, $y = 108$,

$$108 = k \times 3^3$$

$$108 = 27k$$

$$\therefore k = 4$$

$$\therefore y = 4x^3$$

- (ii) When $x = 7$,

$$y = 4 \times 7^3$$

$$= 1372$$

(iii) When $y = 4000$,

$$4000 = 4 \times x^3$$

$$x^3 = \frac{4000}{4}$$

$$= 1000$$

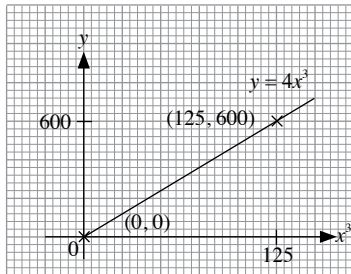
$$x = \sqrt[3]{1000}$$

$$= 10$$

(iv) $y = 4x^3$

When $x = 0$, $y = 0$.

When $x = 5$, $y = 600$.



4. (i) Since n is directly proportional to m^2 ,
then $n = km^2$, where k is a constant.

When $m = 2.5$, $n = 9.375$,

$$9.375 = k \times 2.5^2$$

$$\therefore k = 1.5$$

$$\therefore n = 1.5m^2$$

When $m = 3$,

$$n = 1.5 \times 3^2$$

$$= 13.5$$

(ii) When $n = 181.5$,

$$181.5 = 1.5 \times m^2$$

$$m^2 = \frac{181.5}{1.5}$$

$$= 121$$

$$m = \pm \sqrt{121}$$

$$= \pm 11$$

5. (i) Since t is directly proportional to $\sqrt[3]{s}$,

then $t = k \sqrt[3]{s}$, where k is a constant.

When $s = 64$, $t = 4$,

$$4 = k \times \sqrt[3]{64}$$

$$= 4k$$

$$\therefore k = 1$$

$$\therefore t = \sqrt[3]{s}$$

When $s = 125$,

$$t = \sqrt[3]{125}$$

$$= 5$$

(ii) When $t = 2$,

$$2 = \sqrt[3]{s}$$

$$s = 2^3$$

$$= 8$$

6. (i) Since y is inversely proportional to x ,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 3$, $y = 4$,

$$4 = \frac{k}{3}$$

$$\therefore k = 12$$

$$\therefore y = \frac{12}{x}$$

(ii) When $x = 6$,

$$y = \frac{12}{6}$$

$$= 2$$

(iii) When $y = 24$,

$$24 = \frac{12}{x}$$

$$x = \frac{12}{24}$$

$$= 0.5$$

7. (i) Since q is inversely proportional to p^2 ,

then $q = \frac{k}{p^2}$, where k is a constant.

When $p = 5$, $q = 3$,

$$3 = \frac{k}{5^2}$$

$$\therefore k = 75$$

$$\therefore q = \frac{75}{p^2}$$

(ii) When $p = 10$,

$$q = \frac{75}{10^2}$$

$$= 0.75$$

(iii) When $q = \frac{1}{3}$,

$$\frac{1}{3} = \frac{75}{p^2}$$

$$p^2 = 75 \times 3$$

$$= 225$$

$$p = -\sqrt{225} \quad (p < 0)$$

$$= -15$$

8. (i) Since z is inversely proportional to $w + 3$,

then $z = \frac{k}{w + 3}$, where k is a constant.

When $w = 3$, $z = 4$,

$$4 = \frac{k}{3 + 3}$$

$$\therefore k = 24$$

$$\therefore z = \frac{24}{w + 3}$$

When $w = 9$,

$$z = \frac{24}{9 + 3}$$

$$= 2$$

(ii) When $z = 2.4$,

$$\begin{aligned} 2.4 &= \frac{24}{w+3} \\ w+3 &= \frac{24}{2.4} \\ &= 10 \\ w &= 7 \end{aligned}$$

9.

x	0.2	0.5	1	1.25	2
y	37.5	6	1.5	0.96	0.375

Since y is inversely proportional to $2x^2$,

then $y = \frac{k}{2x^2}$, where k is a constant.

When $x = 2$, $y = 0.375$,

$$\begin{aligned} 0.375 &= \frac{k}{2(2)^2} \\ \therefore k &= 3 \end{aligned}$$

$$\therefore y = \frac{3}{2x^2}$$

When $y = 1.5$,

$$1.5 = \frac{3}{2x^2}$$

$$2x^2 = \frac{3}{1.5}$$

$$= 2$$

$$x^2 = 1$$

$$x = 1 \quad (x > 0)$$

When $y = 0.96$,

$$0.96 = \frac{3}{2x^2}$$

$$2x^2 = \frac{3}{0.96}$$

$$= 3.125$$

$$x^2 = 1.5625$$

$$x = \sqrt{1.5625} \quad (x > 0)$$

$$= 1.25$$

When $x = 0.2$

$$y = \frac{3}{2(0.2)^2}$$

$$= 37.5$$

When $x = 0.5$,

$$y = \frac{3}{2(0.5)^2}$$

$$= 6$$

10. (i) Total monthly charges

$$= \$9.81 + \$0.086 \times 300$$

$$= \$35.61$$

(ii) Variable amount = $\$20.56 - \$9.81$

$$= \$10.75$$

$$\text{Duration of usage} = \frac{10.75}{0.086}$$

$$= 125 \text{ minutes}$$

(iii) Variable amount = $n \times \$0.086$

$$= \$0.086n$$

Total income = variable amount + fixed amount

$$\therefore C = 0.086n + 9.81$$

$$C - 9.81 = 0.086n$$

Since $\frac{C - 9.81}{n} = 0.086$ is a constant, then $C - 9.81$ is directly

proportional to n .

11. (i) Since G is directly proportional to h ,

then $G = kh$, where k is a constant.

When $h = 40$, $G = 2200$,

$$2200 = k \times 40$$

$$\therefore k = 55$$

$$\therefore G = 55h$$

(ii) When $h = 22$,

$$G = 55 \times 22$$

$$= 1210$$

$\therefore$ The gravitational potential energy of the objects is 1210 J.

(iii) When $G = 3025$,

$$3025 = 55 \times h$$

$$h = \frac{3025}{55}$$

$$= 55$$

$\therefore$ The height of the object above the surface of the Earth is 55 m.

12. Let the donations Kate makes be $\$d$,

the savings of Kate be $\$s$.

Since d is directly proportional to s^2 ,

then $d = ks^2$, where k is a constant.

When $s = 900$,

$$d = k \times 900^2$$

$$= 810\,000k$$

When $s = 1200$,

$$d = k \times 1200^2$$

$$= 1\,440\,000k$$

Since Kate's donation increases by $\$35$,

$$1\,440\,000k - 810\,000k = 35$$

$$630\,000k = 35$$

$$k = \frac{35}{630\,000}$$

$$= \frac{1}{18\,000}$$

$$\text{Amount Kate donates in January} = \frac{1}{18\,000} \times 900^2$$

$$= \$45$$

$$\text{Amount Kate donates in February} = \frac{1}{18\,000} \times 1200^2 \text{ or } 45 + 35$$

$$= \$80$$

13. (i) Since P is inversely proportional to V ,

then $P = \frac{k}{V}$, where k is a constant.

When $V = 4000$, $P = 250$,

$$250 = \frac{k}{4000}$$

$$\therefore k = 1\,000\,000$$

$$\therefore P = \frac{1\,000\,000}{V}$$

When $V = 5000$,

$$P = \frac{1\,000\,000}{5000} \\ = 200$$

$\therefore$ The pressure of the gas is 200 Pa.

- (ii) When $P = 125$,

$$125 = \frac{1\,000\,000}{V}$$

$$V = \frac{1\,000\,000}{125} \\ = 8000$$

$\therefore$ The volume of the gas is 8000 dm^3 .

14. Let the number of days for 5 men to complete the job be x .

The number of men is inversely proportional to the number of days to complete the job.

5 men take x days to complete the job.

1 man takes $x \times 5$ days to complete the job.

6 men take $\frac{x \times 5}{6}$ days to complete the job.

Since the job can be completed 8 days earlier when 1 more man is hired,

$$\frac{x \times 5}{6} = x - 8$$

$$\frac{5x}{6} = x - 8$$

$$5x = 6(x - 8)$$

$$= 6x - 48$$

$$x = 48$$

It takes (48×5) days for 1 man to complete the job.

It takes 1 day for $(1 \times 48 \times 5)$ men to complete the job.

It takes $48 - 28 = 20$ days for $\frac{1 \times 48 \times 5}{20} = 12$ men to complete

the job.

$$\text{Additional number of men to hire} = 12 - 5 \\ = 7$$

$\therefore$ 7 more men should be hired.

Challenge Yourself

1. (a) Since A is directly proportional to C ,

then $A = k_1 C$, where k_1 is a constant.

Since B is directly proportional to C ,

then $B = k_2 C$, where k_2 is a constant.

$$A + B = k_1 C + k_2 C$$

$$= (k_1 + k_2)C$$

Since $\frac{A + B}{C} = k_1 + k_2$ is a constant, then $A + B$ is directly proportional to C .

- (b) From (a),

$$A - B = k_1 C - k_2 C$$

$$= (k_1 - k_2)C$$

Since $\frac{A - B}{C} = k_1 - k_2$ is a constant, then $A - B$ is directly proportional to C .

- (c) $AB = (k_1 C)(k_2 C)$

$$= k_1 k_2 C^2$$

$$\sqrt{AB} = \sqrt{k_1 k_2 C^2}$$

$$= \sqrt{k_1 k_2} C$$

Since $\frac{\sqrt{AB}}{C} = \sqrt{k_1 k_2}$ is a constant, then $\sqrt{AB}$ is directly proportional to C .

2. (i) Since z is directly proportional to x^2 and inversely

proportional to $\sqrt{y}$, then $z = \frac{kx^2}{\sqrt{y}}$, where k is a constant.

- (ii) When $x = 2$, $y = 9$, $z = 16$,

$$16 = \frac{k(2)^2}{\sqrt{9}}$$

$$= \frac{4k}{3}$$

$$\therefore k = 16 \times \frac{3}{4}$$

$$= 12$$

$$\therefore z = \frac{12x^2}{\sqrt{y}}$$

When $x = 5$, $y = 4$,

$$z = \frac{12(5)^2}{\sqrt{4}} \\ = 150$$

3. (i) Since T is directly proportional to B and inversely proportional to P , then

$$T = \frac{kB}{P}, \text{ where } k \text{ is a constant.}$$

When $B = 3$, $P = 18$, $T = 20$,

$$20 = \frac{k \times 3}{18}$$

$$= \frac{k}{6}$$

$$\therefore k = 120$$

$$\therefore T = \frac{120B}{P}$$

(ii) When $B = 4$, $P = 16$,

$$T = \frac{120 \times 4}{16}$$
$$= 30$$

$\therefore$ The number of days needed is 30.

(iii) When $B = 10$, $T = 24$,

$$24 = \frac{120 \times 10}{P}$$

$$P = \frac{120 \times 10}{24}$$
$$= 50$$

$\therefore$ 50 painters need to be employed.

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