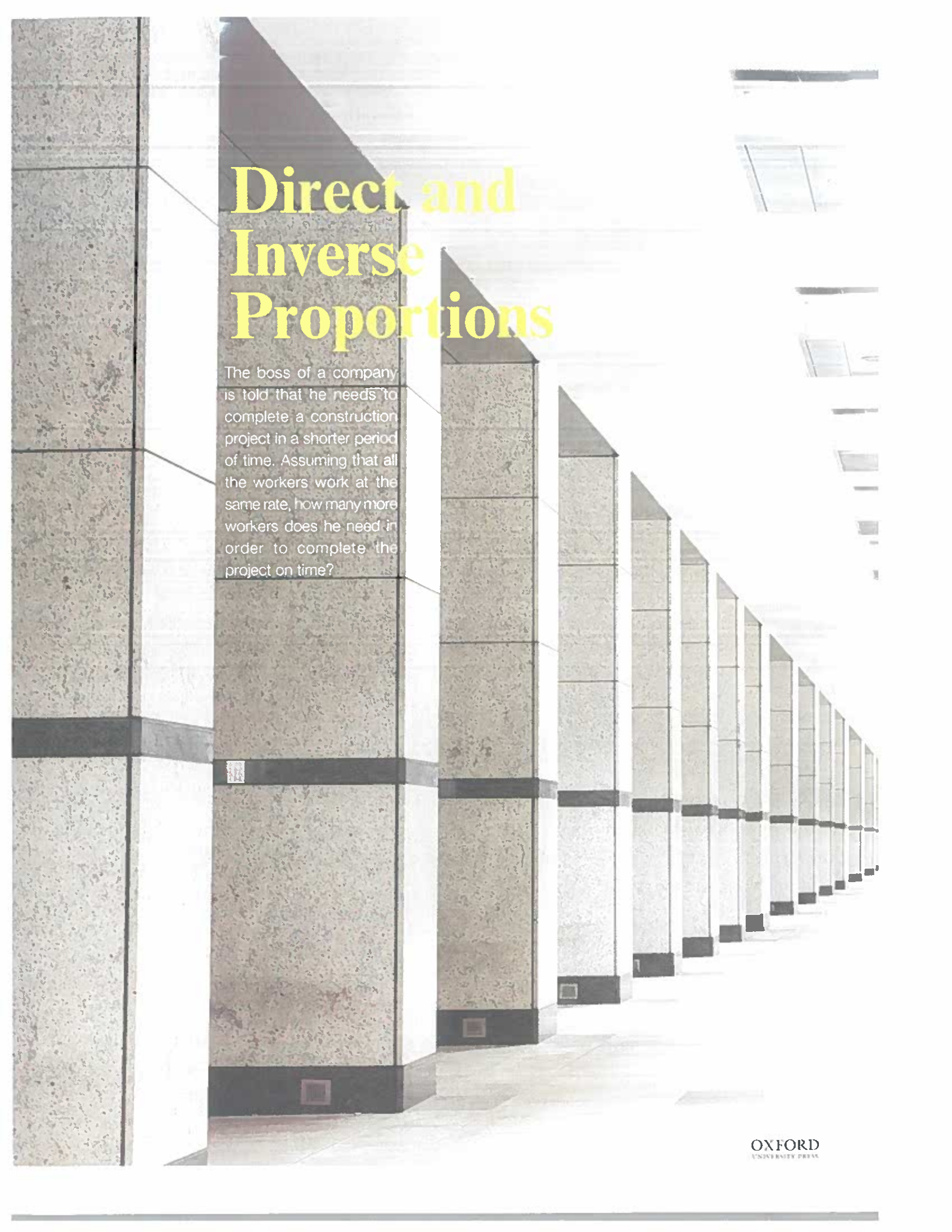
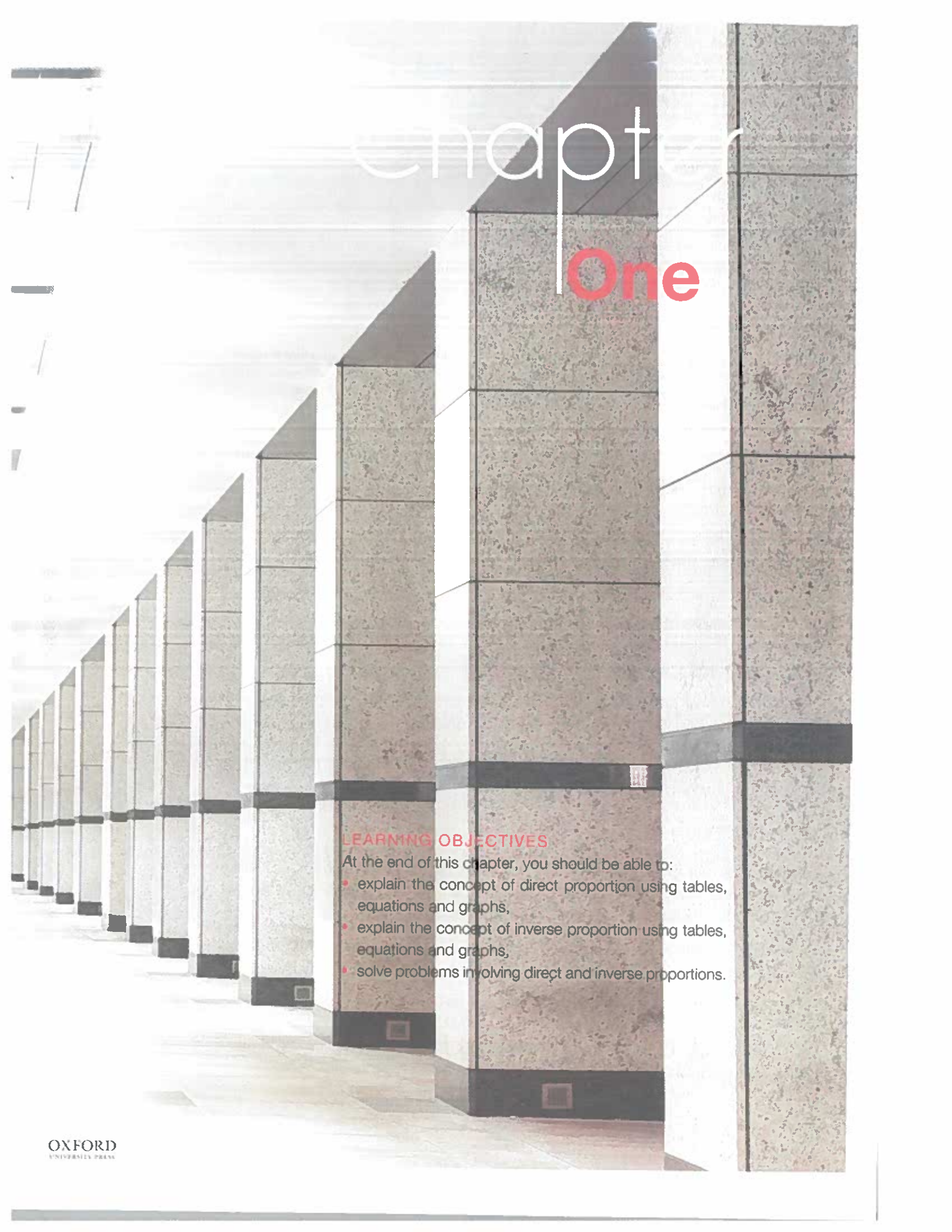




Direct and Inverse Proportions

The boss of a company is told that he needs to complete a construction project in a shorter period of time. Assuming that all the workers work at the same rate, how many more workers does he need in order to complete the project on time?



chapter

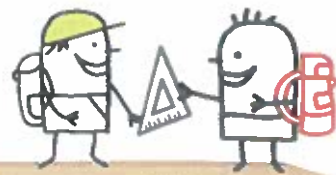
One

LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- explain the concept of direct proportion using tables, equations and graphs,
- explain the concept of inverse proportion using tables, equations and graphs,
- solve problems involving direct and inverse proportions.

1.1 Direct Proportion



Investigation

Direct Proportion

In Singapore, if we borrow books from a public library and are late in returning the books, we will be fined 15 cents per day for each overdue book. Table 1.1 shows the fines for an overdue book.

Number of days (x)	1	2	3	4	5	6	7	8	9	10
Fine (y cents)	15	30	45	60	75	90	105	120	135	150

Table 1.1

1. If the number of days a book is overdue increases, will the fine increase or decrease?
2. If the number of days a book is overdue is doubled, how will the fine change?
Hint: Compare the fines when a book is overdue for 3 days and for 6 days.
3. If the number of days a book is overdue is tripled, what will happen to the fine?
4. If the number of days a book is overdue is halved, how will the fine change?
Hint: Compare the fines when a book is overdue for 10 days and for 5 days.
5. If the number of days a book is overdue is reduced to $\frac{1}{3}$ of the original number, what will happen to the fine?

From the investigation, we notice that as the number of days, x , a book is overdue increases, the fine, y cents, increases *proportionally*, i.e. if x is doubled, then y will be doubled; if x is tripled, then y will be tripled.

Similarly, as the number of days, x , a book is overdue decreases, the fine, y cents, decreases *proportionally*, i.e. if x is halved, then y will be halved; if x is reduced to $\frac{1}{3}$ of its original value, then y will be reduced to $\frac{1}{3}$ of its original value.

This relationship is known as **direct proportion**. We say that the fine, y cents, is *directly proportional* to the number of days, x , a book is overdue.





Class Discussion

Real-Life Examples of Quantities in Direct Proportion

Work in pairs.

1. Give a few more real-life examples of quantities that are in direct proportion.
2. Explain why they are directly proportional to each other.

Complete Table 1.2.

Number of days (x)	1	2	3	4	5	6	7	8	9	10
Fine (y cents)	15	30	45	60	75	90	105	120	135	150
Rate ($\frac{y}{x}$)	$\frac{15}{1} = 15$	$\frac{30}{2} = 15$	$\frac{45}{3} = 15$							

Table 1.2

What can we observe about the rate $\frac{y}{x}$?

What does $\frac{y}{x}$ represent? What does the constant '15' mean in this context?

In direct proportion, the rate $\frac{y}{x}$ is a **constant**. In this case, $\frac{y}{x} = 15$.

Let the number of days a book is overdue be $x_1 = 3$. Then the corresponding fine is $y_1 = 45$.

Let the number of days a book is overdue be $x_2 = 6$. Then the corresponding fine is $y_2 = 90$.

From Table 1.2, $\frac{y_1}{x_1} = \frac{45}{3} = 15$ and $\frac{y_2}{x_2} = \frac{90}{6} = 15$.

$\therefore \frac{y_1}{x_1} = \frac{y_2}{x_2} = 15$ (constant), i.e. the two **rates** $\frac{y_1}{x_1}$ and $\frac{y_2}{x_2}$ are equal

Moreover, $\frac{x_2}{x_1} = \frac{6}{3} = 2$ and $\frac{y_2}{y_1} = \frac{90}{45} = 2$, i.e. y is doubled when x is doubled.

$\therefore \frac{x_2}{x_1} = \frac{y_2}{y_1}$, i.e. the two **ratios** $\frac{x_2}{x_1}$ and $\frac{y_2}{y_1}$ are equal (This can also be obtained by rearranging the equation

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}.)$$

To conclude, we have:

If y is **directly proportional** to x , then $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ or $\frac{x_2}{x_1} = \frac{y_2}{y_1}$.

ATTENTION

In Book 1, we have learnt that a **rate** compares two or more quantities of **different kinds**.

In this case, $\frac{y_1}{x_1}$ (or $\frac{y_2}{x_2}$) compares the fine with the number of days a book is overdue. We have also learnt that a **ratio** compares two or more quantities of the **same kind**. In this case, $\frac{x_2}{x_1}$ (or $x_2 : x_1$) is a comparison of the number of days a book is overdue and $\frac{y_2}{y_1}$ (or $y_2 : y_1$) is a comparison of the fines.

Worked Example 1

(Simple Problem involving Direct Proportion)

If 6 kg of biscuits cost \$27, calculate the cost of 13 kg of biscuits.

Solution:

First, we note that the cost of the biscuits is directly proportional to the mass of the biscuits.

Method 1: Unitary Method

6 kg of biscuits cost \$27.

1 kg of biscuits costs $\frac{\$27}{6}$.

13 kg of biscuits cost $\frac{\$27}{6} \times 13 = \$58.50$.

Method 2: Proportion Method

Let the cost of 13 kg of biscuits be \$ x .

$$\text{Then } \frac{x}{13} = \frac{27}{6} \quad \left(\frac{x_1}{y_1} = \frac{x_2}{y_2} \right)$$

$$\begin{aligned} x &= \frac{27}{6} \times 13 \\ &= \$58.50 \end{aligned}$$

Alternatively,

$$\frac{x}{27} = \frac{13}{6} \quad \left(\frac{x_1}{x_2} = \frac{y_1}{y_2} \right)$$

$$\begin{aligned} x &= \frac{13}{6} \times 27 \\ &= \$58.50 \end{aligned}$$

$\therefore$ 13 kg of biscuits cost \$58.50.

INFORMATION

Method 1 is called the *unitary* method because it involves finding the cost of 1 kg (or 1 *unit*) of biscuits first.



PRACTISE NOW 1

- (a) If 50 g of sweets cost \$2.10, find the cost of 380 g of sweets, giving your answer correct to the nearest 5 cents.
- (b) $\frac{3}{4}$ of a piece of metal has a mass of 15 kg. What is the mass of $\frac{2}{5}$ of the piece of metal?

SIMILAR QUESTIONS

Exercise 1A Questions 1–2,
5(a)–(b), 6

1.2 Algebraic and Graphical Representations of Direct Proportion



In the example on overdue books in Section 1.1, we have found that $\frac{y}{x} = 15$, which is a constant. If we represent this constant by k , then $\frac{y}{x} = k$ or $y = kx$, where $k \neq 0$. Hence, we have:

If y is *directly proportional* to x , then $\frac{y}{x} = k$ or $y = kx$, where k is a constant and $k \neq 0$.



If we substitute $k = 0$ into $y = kx$, what can we say about the relationship between x and y ?



Graphical Representation of Direct Proportion

Consider the example on overdue books in Section 1.1. Table 1.3 shows the fines, y cents, for various number of days, x , a book is overdue, where $\frac{y}{x} = 15$ or $y = 15x$. What does $y = 15x$ mean in this context?

Number of days (x)	0	1	2	3	4	5	6	7	8	9	10
Fine (y cents)	0	15	30	45	60	75	90	105	120	135	150

Table 1.3



1. Plot the graph of y against x in Fig. 1.1.

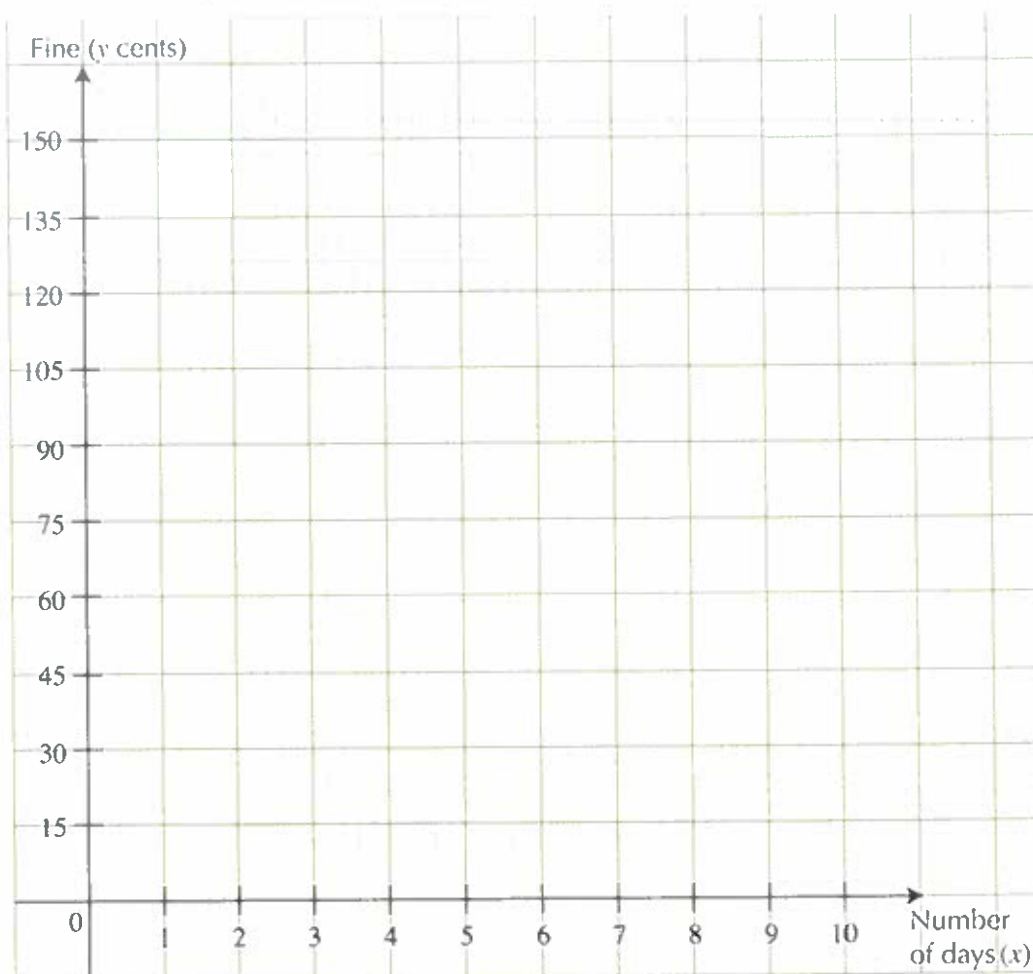


Fig. 1.1

2. What is the shape of the graph?
3. Does the graph pass through the origin?

To conclude, we have:

If y is *directly proportional* to x , then the graph of y against x is a straight line that passes through the origin.



- Thinking Time**
1. If y is directly proportional to x , is x directly proportional to y ? Explain your answer.
 2. Suppose y is directly proportional to x . If we plot the graph of x against y , will we get a straight line that passes through the origin? Explain your answer.
 3. If y is directly proportional to x , then the graph of y against x passes through the origin. If the graph of y against x does not pass through the origin, is y directly proportional to x ? Explain your answer.
 4. As x increases, y also increases. Can we conclude that y is directly proportional to x ? Explain your answer.

Worked Example 2

Equation of Direct Proportion

If y is directly proportional to x and $y = 12$ when $x = 4$, find

- an equation connecting x and y ,
- the value of y when $x = 8$,
- the value of x when $y = 21$.

Solution:

- (i) Since y is directly proportional to x , then $y = kx$, where k is a constant.

When $x = 4$, $y = 12$,

$$12 = k \times 4$$

$$\therefore k = 3$$

$$\therefore y = 3x$$

- (ii) When $x = 8$,

$$y = 3 \times 8$$

$$= 24$$

Alternatively,

when $x = 8$, x is double(1)

$y = 2 \times 12$ (y is double(1))

$$= 24$$

We can also use $\frac{y_2}{y_1} = \frac{x_2}{x_1}$.

$$\text{i.e. } \frac{y}{12} = \frac{8}{4}$$

$$y = 2 \times 12$$

$$= 24$$

- (iii) Substitute $y = 21$ into $y = 3x$:

$$21 = 3x$$

$$\therefore x = \frac{21}{3}$$

$$= 7$$

ATTENTION

Since y is directly proportional to x , then $\frac{y}{x} = k$ or $y = kx$, where k is a constant and $k \neq 0$.

PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 1A Questions 3–4, 7–8, 9(a)–(b)

- If y is directly proportional to x and $y = 10$ when $x = 2$, find
 - an equation connecting x and y ,
 - the value of y when $x = 10$,
 - the value of x when $y = 60$.
- If y is directly proportional to x and $y = 5$ when $x = 2$, find the value of y when $x = 7$.
- Given that y is directly proportional to x , copy and complete the table.

x	4	5	7		
y		30		48	57

Worked Example 3

PROBLEM INVOLVING DIRECT PROPORTION

The expenses, \$ E , of a tea party are directly proportional to the number of guests, N , present. When there are 30 guests present at the tea party, the expenses incurred are \$210.

- Find an equation connecting E and N .
- Calculate the expenses incurred when there are 80 guests present at the tea party.
- Draw the graph of E against N .

Solution:

- (i) Since E is directly proportional to N ,
then $E = kN$, where k is a constant.

When $N = 30$, $E = 210$,

$$210 = k \times 30$$

$$\therefore k = 7$$

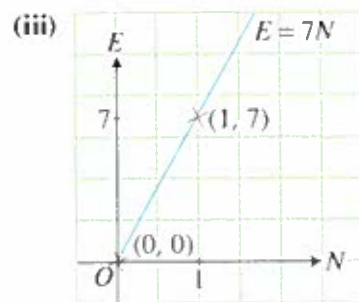
$$\therefore E = 7N$$

- (ii) When $N = 80$,

$$E = 7 \times 80$$

$$= 560$$

$\therefore$ The expenses incurred are \$560.



$$E = 7N$$

When $N = 0$, $E = 0$.

When $N = 1$, $E = 7$.

ATTENTION

Worked Example 3 can also be solved using the Unitary Method or the Proportion Method (see Worked Example 1). However, as there are forms of direct proportion where these two methods will not work, we need to learn the algebraic method to solve them (see Worked Example 6).

PRACTISE NOW 3

The cost, \$ C , of transporting goods is directly proportional to the distance covered, d km. The cost of transporting goods over a distance of 60 km is \$100.

- Find an equation connecting C and d .
- Find the cost of transporting goods over a distance of 45 km.
- If the cost of transporting goods is \$120, calculate the distance covered.
- Draw the graph of C against d .

SIMILAR QUESTIONS

Exercise 1A Questions 10–14

Worked Example 4

(Non-Example of Direct Proportion)

The total monthly charges, \$ C , for a mobile plan consist of a fixed amount of \$20 and a variable amount which depends on the usage. For every minute used, \$0.20 is charged.

- If the duration of usage is 120 minutes, calculate the total monthly charges for the mobile plan.
- If the total monthly charges for the mobile plan are \$50, find the duration of usage.
- Write down a formula connecting C and n , where n is the number of minutes of usage.
- Draw the graph of C against n . Is C directly proportional to n ? Use your graph to explain your answer.

Solution:

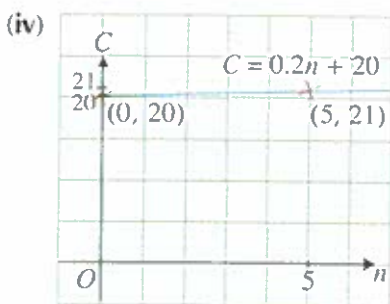
(i) Total monthly charges for the mobile plan = $\$20 + 120 \times \$0.20$
 $= \$44$

(ii) Variable amount = $\$50 - \$20$
 $= \$30$

Duration of usage = $\frac{30}{0.20}$
 $= 150 \text{ minutes}$

(iii) Variable amount = $n \times \$0.20$
 $= \$0.2n$

Total monthly charges = variable amount + fixed amount
 $\therefore C = 0.2n + 20$



$$C = 0.2n + 20$$

When $n = 0$, $C = 20$.

When $n = 5$, $C = 21$.

C is **not** directly proportional to n because the line does not pass through the origin.

ATTENTION

- Since n cannot be negative, the line must start from $n = 0$.
- When $n = 120$, $C = 44$.
When $n = 150$, $C = 50$.
Since $\frac{44}{120} \neq \frac{50}{150} = \frac{C}{n}$ is not a constant,
 $\therefore C$ is not directly proportional to n .

PRACTISE NOW 4

The total monthly cost, \$ C , of running a kindergarten consists of a fixed amount of \$5000 and a variable amount which depends on the enrolment. For every child enrolled, the monthly cost increases by \$41.

- If the enrolment is 200, find the total monthly cost of running the kindergarten.
- If the total monthly cost of running the kindergarten is \$20 580, calculate the number of children enrolled in the kindergarten.
- Write down a formula connecting C and n , where n is the number of children enrolled in the kindergarten.
- Draw the graph of C against n . Is C directly proportional to n ? Use your graph to explain your answer.

SIMILAR QUESTIONS

Exercise 1A Questions 15–16



Exercise 1A

BASIC LEVEL

- 108 identical books have a mass of 30 kg. Find
 - the mass of 150 such books,
 - the number of such books that have a mass of 20 kg.
- In a bookstore, 60 identical books occupy a length of 1.5 m on a shelf. Find
 - the length occupied by 50 such books on a shelf,
 - the number of such books needed to completely occupy a shelf that is 80 cm long.
- If x is directly proportional to y and $x = 4.5$ when $y = 3$, find
 - an equation connecting x and y ,
 - the value of x when $y = 6$,
 - the value of y when $x = 12$.
- If Q is directly proportional to P and $Q = 28$ when $P = 4$,
 - express Q in terms of P ,
 - find the value of Q when $P = 5$,
 - calculate the value of P when $Q = 42$.

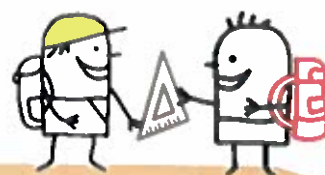
INTERMEDIATE LEVEL

- Find the cost of
 - 10 kg of tea leaves when 3 kg of tea leaves cost \$18,
 - a kg of sugar when b kg of sugar cost \$ c .
- $\frac{5}{9}$ of a piece of metal has a mass of 7 kg. What is the mass of $\frac{2}{7}$ of the piece of metal?
- If z is directly proportional to x and $z = 12$ when $x = 3$, find the value of x when $z = 18$.
- If B is directly proportional to A and $B = 3$ when $A = 18$, find the value of B when $A = 24$.
- For each of the following, y is directly proportional to x . Copy and complete the tables.
 - | | | | | | |
|-----|---|----|----|---|----|
| x | 4 | 20 | 24 | | |
| y | | | 6 | 9 | 11 |
 - | | | | | | |
|-----|---|-----|-----|-----|------|
| x | 2 | 3 | 5.5 | | |
| y | | 3.6 | | 9.6 | 11.4 |
- If y is directly proportional to x and $y = 20$ when $x = 5$,
 - find an equation connecting x and y ,
 - draw the graph of y against x .
- If z is directly proportional to y and $z = 48$ when $y = 6$,
 - find an equation connecting y and z ,
 - draw the graph of z against y .
- The net force, F newtons, needed to push a block along a horizontal surface is directly proportional to the mass, m kg, of the block. When $m = 5$, $F = 49$.
 - Find an equation connecting F and m .
 - Find the value of F when $m = 14$.
 - Calculate the value of m when $F = 215.6$.
 - Draw the graph of F against m .
- The pressure, P pascals, of a gas in a container is directly proportional to its temperature, T kelvin. When $T = 10$, $P = 25$.
 - Find an equation connecting P and T .
 - Find the value of P when $T = 24$.
 - Calculate the value of T when $P = 12$.
 - Draw the graph of P against T .
- The amount of voltage, V volts, needed to send a fixed amount of current through a wire, is directly proportional to its resistance, R ohms. When $R = 6$, $V = 9$.
 - Find an equation connecting V and R .
 - Find the value of V when $R = 15$.
 - Calculate the value of R when $V = 15$.
 - Draw the graph of V against R .

15. The total monthly income, $\$D$, of a salesman who sells tyres consists of a basic salary of $\$600$ and a variable amount which depends on the number of tyres he sells. For each tyre he sells, he receives $\$8$.
- If the salesman sells 95 tyres in a particular month, find his total income for that month.
 - If the salesman's monthly income for a particular month is $\$1680$, calculate the number of tyres he sells in that month.
 - Write down a formula connecting D and n , where n is the number of tyres the salesman sells in a month.
 - Draw the graph of D against n . Is D directly proportional to n ? Use your graph to explain your answer.
16. A machine which manufactures ice needs to be run for 10 minutes to warm up *before* the production of ice begins. The mass, in tonnes, of ice produced is directly proportional to the number of hours of production. Given that 20 tonnes of ice are produced when the machine runs for half an hour, find the mass of ice manufactured when the machine runs for 1.75 hours.



1.3 Other Forms of Direct Proportion



Investigation

Other Forms of Direct Proportion

The variables x and y are connected by the equation $y = 3x^2$.

- Some values of x , and the corresponding values of y and $\frac{y}{x}$ are given in Table 1.4.

x	1	2	3	4
y	3	12	27	48
$\frac{y}{x}$	3	6	9	12

Table 1.4

Fig. 1.2 shows the graph of y against x .

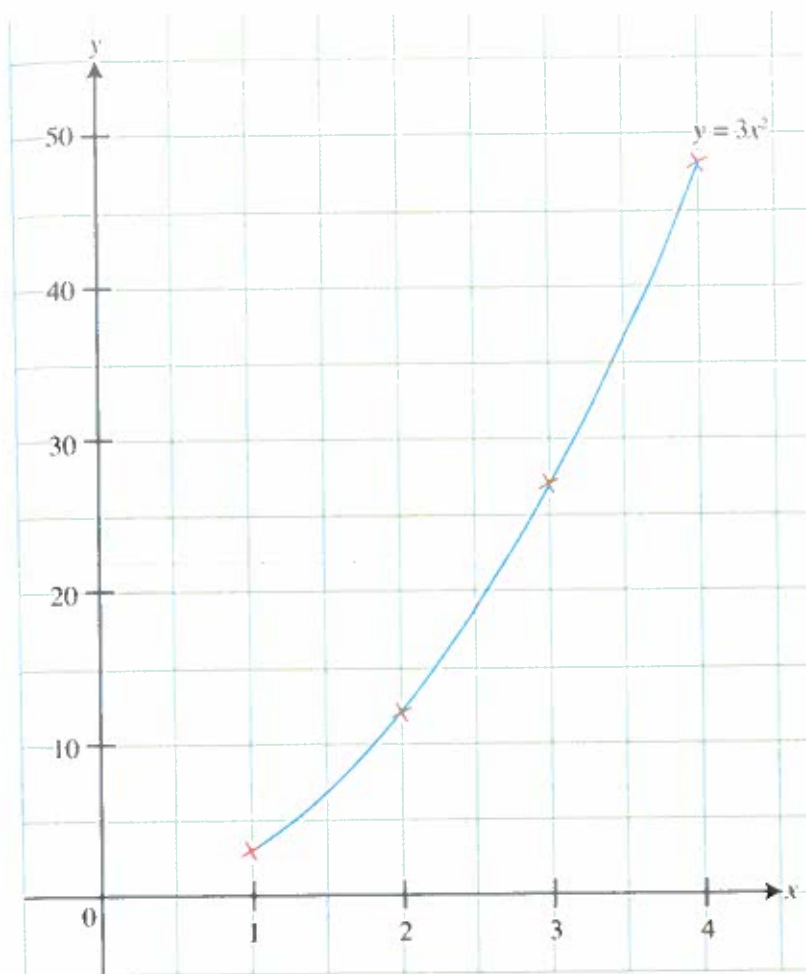


Fig. 1.2

Is y directly proportional to x ? Explain your answer.

2. Now, let us plot the graph of y against x^2 .

Some values of x , and the corresponding values of x^2 , y and $\frac{y}{x^2}$ are given in Table 1.5.

x	1	2	3	4
x^2	1	4	9	16
y	3	12	27	48
$\frac{y}{x^2}$	3	3	3	3

Table 1.5

Plot the graph of y against x^2 in Fig. 1.3. The first three points have been plotted for you.

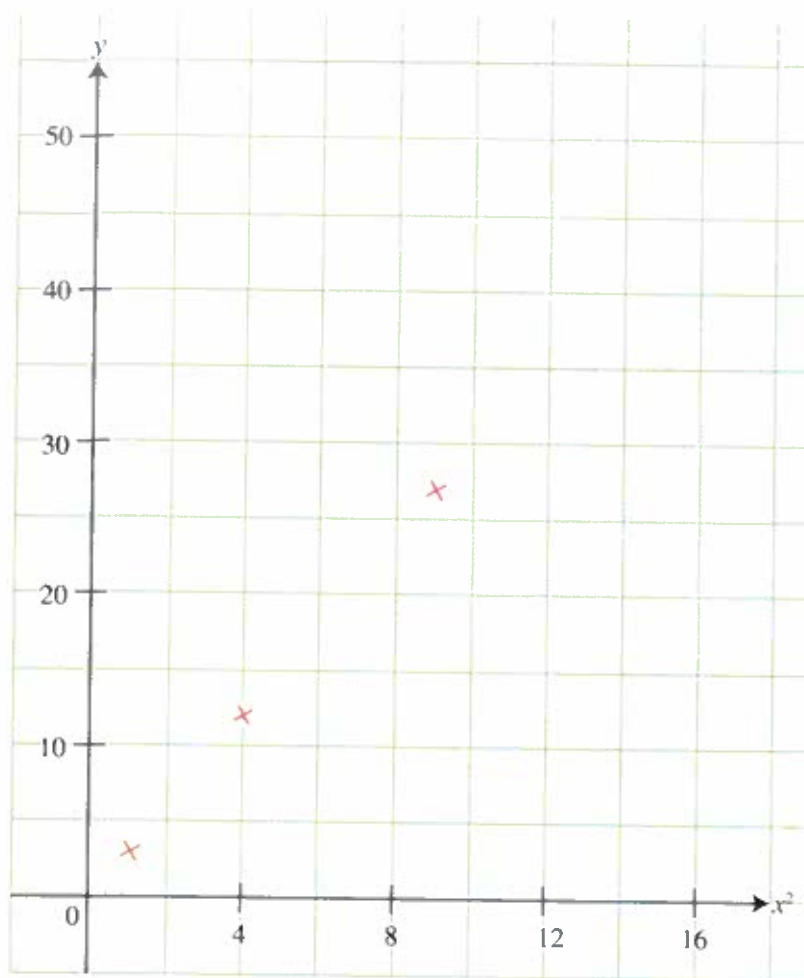


Fig. 1.3

Is y directly proportional to x^2 ? Explain your answer.

From the investigation, when $y = 3x^2$, y is not directly proportional to x because

$\frac{y}{x} = 3x$ is not a constant. However, y is directly proportional to x^2 because $\frac{y}{x^2} = 3$ is constant.

INFORMATION

Another way to look at this is to let $X = x^2$ such that $y = 3x^2$ becomes $y = 3X$, i.e. $\frac{y}{X} = 3$. Therefore, y is directly proportional to X ($= x^2$).

Worked Example 5

Identify the variables which are directly proportional to each other.

For each of the following equations, state the two variables which are directly proportional to each other and explain your answer.

(a) $y = 5x^3$

(b) $y^2 = \sqrt{x}$

Solution:

(a) Since $y = 5x^3$, i.e. $\frac{y}{x^3} = 5$ is a constant, then y is directly proportional to x^3 .

(b) Since $y^2 = \sqrt{x}$, i.e. $\frac{y^2}{\sqrt{x}} = 1$ is a constant, then y^2 is directly proportional to $\sqrt{x}$.

PRACTISE NOW 5

For each of the following equations, state the two variables which are directly proportional to each other and explain your answer.

(a) $y = 6x^2$

(b) $\sqrt{y} = x^3$

SIMILAR QUESTIONS

Exercise 1B Questions 4(a)–(d)



- In Worked Example 4, we have found that the formula connecting C and n is $C = 0.2n + 20$ and that C is not directly proportional to n . However, n is directly proportional to a variable. What is this variable?
- In the equation $y - 1 = 4x$, state the two variables which are directly proportional to each other and explain your answer.

Worked Example 6

Equation of Another Form of Direct Proportion

If y is directly proportional to x^2 and $y = 20$ when $x = 2$,

- find an equation connecting x and y ,
- calculate the value of y when $x = 3$,
- find the values of x when $y = 1.25$,
- draw the graph of y against x^2 .

Solution:

- (i) Since y is directly proportional to x^2 ,
then $y = kx^2$, where k is a constant.

When $x = 2$, $y = 20$,

$$20 = k \times 2^2$$

$$20 = 4k$$

$$\therefore k = 5$$

$$\therefore y = 5x^2$$

- (ii) When $x = 3$,

$$y = 5 \times 3^2$$

$$= 45$$

- (iii) When $y = 1.25$,

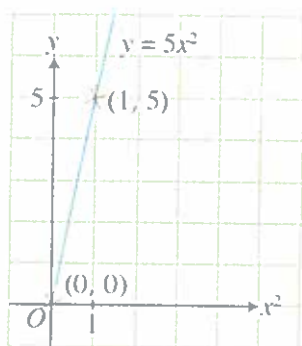
$$1.25 = 5x^2$$

$$x^2 = 0.25$$

$$\therefore x = \pm\sqrt{0.25}$$

$$= \pm 0.5$$

- (iv) Since y is directly proportional to x^2 , then the graph of y against x^2 is a straight line that passes through the origin.



$$y = 5x^2$$

When $x = 0$, $y = 0$.

When $x = 1$, $y = 5$.

ATTENTION

It is difficult to use the Unitary Method or the Proportion Method (see Worked Example 1) to solve direct proportion problems like in Worked Example 6. Thus we need to learn the algebraic method (see Worked Example 3).

ATTENTION

As x^2 cannot be negative, the line must start from the origin where $x^2 = 0$.

PRACTISE NOW 6

1. If y is directly proportional to x^2 and $y = 18$ when $x = 3$,

(i) find an equation connecting x and y ,

(ii) find the value of y when $x = 5$,

(iii) calculate the values of x when $y = 32$,

(iv) draw the graph of y against x^2 .

2. If y is directly proportional to x^2 and $y = 21$ when $x = 2$, find the value of y when $x = 4$.

3. Given that y is directly proportional to x^2 , where x is a positive real number, copy and complete the table.

x	2		3	5	
y		56.25	81		441

SIMILAR QUESTIONS

Exercise 1B Questions 1–3, 5–8, 11

Worked Example 7

Problem Involving Another Example of Direct Proportion

The volume, $V \text{ cm}^3$, of a solid is directly proportional to the cube of its radius, $r \text{ cm}$. When the radius of the solid is 6 cm, its volume is 905 cm^3 .

- Find an equation connecting V and r .
- Calculate the volume of the solid when its radius is 10 cm.

Solution:

- (i) Since V is directly proportional to r^3 ,
then $V = kr^3$, where k is a constant.

When $r = 6$, $V = 905$,

$$905 = k \times 6^3$$

$$\therefore k = \frac{905}{216}$$

$$\therefore V = \frac{905}{216} r^3$$

- (ii) When $r = 10$,

$$V = \frac{905}{216} \times 10^3$$

$$= 4189 \frac{22}{27}$$

$\therefore$ The volume of the solid is $4189 \frac{22}{27} \text{ cm}^3$.

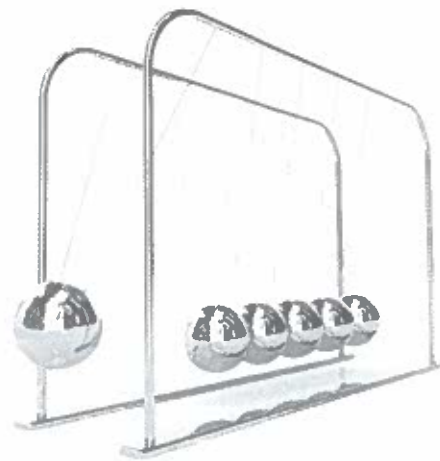
PRACTISE NOW 7

The length, $l \text{ cm}$, of a simple pendulum is directly proportional to the square of its period (time taken to complete one oscillation), T seconds. A pendulum with a length of 220.5 cm has a period of 3 s.

- Find an equation connecting l and T .
- Find the length of a pendulum which has a period of 5 s.
- What is the period of a pendulum which has a length of 0.98 m?

SIMILAR QUESTIONS

Exercise 1B Questions 9–10, 12





Exercise 1B

BASIC LEVEL

- If x is directly proportional to y^3 and $x = 32$ when $y = 2$,
 - find an equation connecting x and y ,
 - find the value of x when $y = 6$,
 - calculate the value of y when $x = 108$,
 - draw the graph of x against y^3 .
- If z^2 is directly proportional to w and $z = 4$ when $w = 8$,
 - find an equation connecting w and z ,
 - find the values of z when $w = 18$,
 - calculate the value of w when $z = 5$,
 - draw the graph of z^2 against w .
- It is given that y is directly proportional to x^n . Write down the value of n when
 - $y \text{ m}^2$ is the area of a square of length $x \text{ m}$,
 - $y \text{ cm}^3$ is the volume of a cube of length $x \text{ cm}$.

INTERMEDIATE LEVEL

- For each of the following equations, state the two variables which are directly proportional to each other and explain your answer.
 - $y = 4x^2$
 - $y = 3\sqrt{x}$
 - $y^2 = 5x^3$
 - $p^3 = q^2$
- If z^2 is directly proportional to x^3 and $z = 8$ when $x = 4$, find the values of z when $x = 9$.
- If q is directly proportional to $(p - 1)^2$ and $q = 20$ when $p = 3$, find the values of p when $q = 80$.
- Given that y is directly proportional to x^3 , copy and complete the table.

x	3	4		6	
y			375	648	1029

- Given that the mass, $m \text{ g}$, of a sphere is directly proportional to the cube of its radius, $r \text{ cm}$, copy and complete the table.

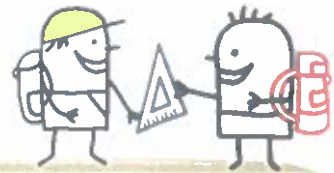
r	0.2		0.7	1.5	
m		0.25		6.75	11.664

- During a certain period in the life of an earthworm, its length, $L \text{ cm}$, is directly proportional to the square root of N , where N is the number of hours after its birth. One hour after an earthworm is born, its length is 2.5 cm .
 - Find an equation connecting L and N .
 - Find the length of an earthworm 4 hours after its birth.
 - How long will it take for an earthworm to grow to a length of 15 cm ?
- If y is directly proportional to x^2 and the difference in the values of y when $x = 1$ and $x = 3$ is 32, find the value of y when $x = -2$.

ADVANCED LEVEL

- y is directly proportional to x^2 and $y = a$ for a particular value of x . Find an expression for y in terms of a , when this value of x is doubled.
- The braking distance of a vehicle is directly proportional to the square of its speed. When the speed of the vehicle is $b \text{ m/s}$, its braking distance is $d \text{ m}$. If the speed of the vehicle is increased by 200%, find the percentage increase in its braking distance.

1.4 Inverse Proportion



Investigation

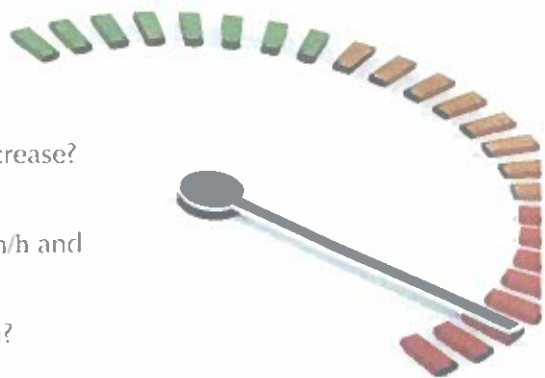
Inverse Proportion

Table 1.6 shows the time taken for a car to travel a distance of 120 km at different speeds.

Speed (x km/h)	10	20	30	40	60	120
Time taken (y hours)	12	6	4	3	2	1

Table 1.6

1. If the speed of the car increases, will the time taken increase or decrease?
2. If the speed of the car is doubled, how will the time taken change?
Hint: Compare the time taken when the speeds of the car are 20 km/h and 40 km/h.
3. If the speed of the car is tripled, what will happen to the time taken?
4. If the speed of the car is halved, how will the time taken change?
Hint: Compare the time taken when the speeds of the car are 60 km/h and 30 km/h.
5. If the speed of the car is reduced to $\frac{1}{3}$ of its original speed, what will happen to the time taken?



From the investigation, we notice that as the speed of the car, x km/h, increases, the time taken, y hours, decreases *proportionally*, i.e. if x is doubled, then y will be halved; if x is tripled, then y will be reduced to $\frac{1}{3}$ of its original value.

Similarly, as the speed of the car, x km/h, decreases, the time taken, y hours, increases *proportionally*, i.e. if x is halved, then y will be doubled; if x is reduced to $\frac{1}{3}$ of its original value, then y will be tripled.

This relationship is known as **inverse proportion**. We say that the speed of the car, x km/h, is *inversely proportional* to the time taken, y hours.



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Class Discussion

Real-Life Examples of Quantities in Inverse Proportion

Work in pairs.

1. Give a few more real-life examples of quantities that are in inverse proportion.
2. Explain why they are inversely proportional to each other.

SIMILAR QUESTIONS

Exercise 1C Questions 1(a)–(e)

Complete Table 1.7.

Speed (x km/h)	10	20	30	40	60	120
Time taken (y hours)	12	6	4	3	2	1
Product xy km	$10 \times 12 = 120$	$20 \times 6 = 120$				

Table 1.7

What can we observe about the product xy ?

In inverse proportion, the product xy is a *constant*. In this case, $xy = 120$ = distance travelled.

Let the speed of the car be $x_1 = 20$. Then the corresponding time taken is $y_1 = 6$.

Let the speed of the car be $x_2 = 40$. Then the corresponding time taken is $y_2 = 3$.

From Table 1.7, $x_1y_1 = 20 \times 6 = 120$ and $x_2y_2 = 40 \times 3 = 120$.

$$\therefore x_1y_1 = x_2y_2 = 120 \text{ (constant)}$$

Rearranging,

$$\frac{y_2}{y_1} = \frac{x_1}{x_2}$$

To conclude, we have:

If y is *inversely proportional* to x , then $\frac{y_2}{y_1} = \frac{x_1}{x_2}$ or $x_1y_1 = x_2y_2$.

ATTENTION

We have learnt that for direct proportion, $\frac{y_2}{y_1} = \frac{x_2}{x_1}$, but for inverse proportion, we have $\frac{y_2}{y_1} = \frac{x_1}{x_2}$ or $x_1y_1 = x_2y_2$. Note the order of x_1 and x_2 .

Worked Example 8

Problem involving Inverse Proportion

10 identical taps can fill a tank in 4 hours. Calculate the time taken for 8 such taps to fill the same tank.

Solution:

First, we note that the time taken to fill the tank is inversely proportional to the number of taps used because as the number of taps increases, the time taken to fill the tank decreases.

Method 1: Unitary Method

10 taps can fill the tank in 4 hours.

1 tap can fill the tank in (10×4) hours. (fewer taps require more time)

8 taps can fill the tank in $\frac{10 \times 4}{8}$ (more taps require less time)
 $= 5$ hours.

Method 2: Proportion Method

Let the time taken for 8 taps to fill the tank be y hours.

Then $8y = 10 \times 4$. ($x_1y_1 = x_2y_2$)

$$y = \frac{10 \times 4}{8}$$

$$= 5$$

$\therefore$ 8 taps can fill the tank in 5 hours.

Problem Solving Tip



Each tap can fill $\frac{1}{10}$ of the tank in 4 hours.

Problem Solving Tip

It is more confusing to use

$\frac{y_2}{y_1} = \frac{x_1}{x_2}$ to solve inverse proportion problems like in Worked Example 8 because we need to switch the order of x_1 and x_2 .

PRACTISE NOW 8

Four identical taps can fill a tank in 70 minutes. Find the time taken for 7 such taps to fill the same tank.

SIMILAR QUESTIONS

Exercise 1C Questions 2, 5–7, 15

Worked Example 9

Problem involving Direct and Inverse Proportions:

5 men can paint 2 identical houses in 3 days. Assuming that all the men work at the same rate, how long will it take 10 men to paint 8 such houses?

Solution:

The three variables are 'number of men', 'number of houses' and 'number of days'.

First, we keep the number of houses constant.

<u>Number of men</u>	<u>Number of houses</u>	<u>Number of days</u>	
5	2	3	
1	2	5×3	(fewer men require more days)
10	2	$\frac{5 \times 3}{10} = 1.5$	(more men require fewer days)

Next, we keep the number of men constant.

<u>Number of men</u>	<u>Number of houses</u>	<u>Number of days</u>	
10	2	1.5	
10	1	$\frac{1.5}{2}$	(fewer houses require fewer days)
10	8	$8 \times \frac{1.5}{2} = 6$	(more houses require more days)

$\therefore$ 10 men will take 6 days to paint 8 houses.

Problem Solving Tip

When three variables are involved, we keep one variable constant at a time.

ATTENTION

The number of days required to paint the houses is inversely proportional to the number of men.

ATTENTION

The number of days required to paint the houses is directly proportional to the number of houses that need to be painted.

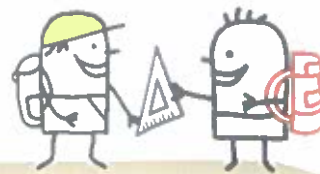
PRACTISE NOW 9

- 3 men can dig 2 identical trenches in 5 hours. Assuming that all the men work at the same rate, how long will it take 5 men to dig 7 such trenches?
- 7 identical taps can fill 3 identical tanks in 45 minutes. How long will it take 5 of the taps to fill one of the tanks?

SIMILAR QUESTIONS

Exercise 1C Questions 13–14, 16

1.5 Algebraic and Graphical Representations of Inverse Proportion



In the investigation on page 19 of Section 1.4, we have found that $xy = 120$, which is a constant. If we represent this constant by k , then $xy = k$ or $y = \frac{k}{x}$, where $k \neq 0$. Hence, we have:

If y is *inversely proportional* to x , then $xy = k$ or $y = \frac{k}{x}$, where k is a constant and $k \neq 0$.



Thinking Time

If we substitute $k = 0$ into $y = \frac{k}{x}$, what can we say about the relationship between x and y ?



Investigation

Graphical Representation of Inverse Proportion

Consider the example on the car in Section 1.4. Table 1.8 shows the time taken, y hours, for the car to travel a distance of 120 km at different speeds, x km/h, where $xy = 120$ or $y = \frac{120}{x}$.

Speed (x km/h)	10	20	30	40	50	60	70	80	90	100	110	120
Time taken (y hours)	12	6	4	3	2.4	2	1.7	1.5	1.3	1.2	1.1	1

Table 1.8

1. If we plot the graph of y against x , what type of graph would we obtain?
2. Plot the graph of y against x in Fig. 1.4.

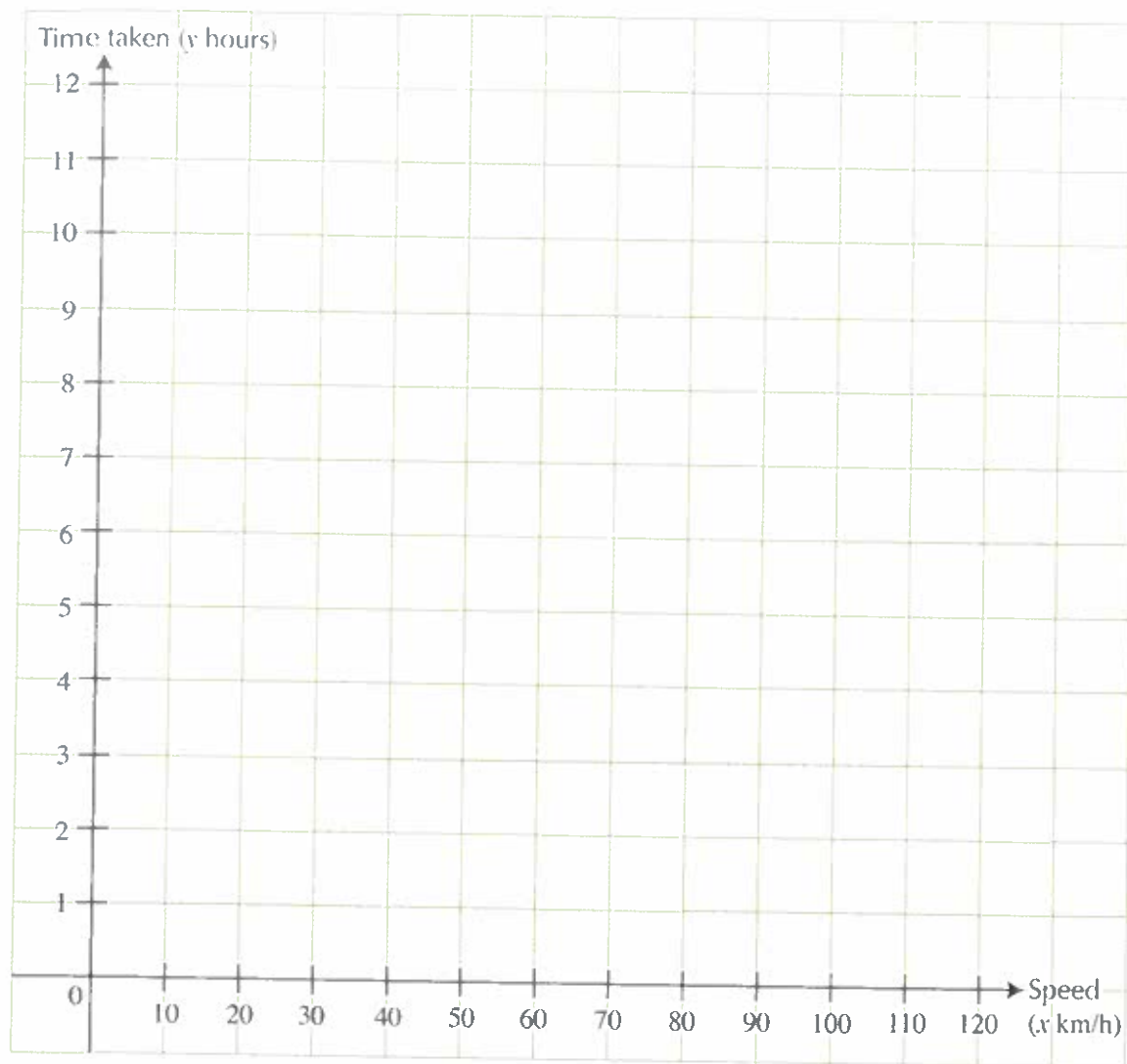


Fig. 1.4

3. If the value of x is doubled, how will the value of y change?

Illustrate this with an example by choosing two appropriate points on the graph you have drawn in Fig. 1.4.

What if we plot the graph of y against $\frac{1}{x}$? How would the graph look like?

Let $X = \frac{1}{x}$. Some values of x , and the corresponding values of X and y are given in Table 1.9. Complete the table.

Speed (x km/h)	10	20	30	40	50	60	70	80	90	100	110	120
$X = \frac{1}{x}$	0.1	0.05	0.033		0.02	0.017	0.014	0.013	0.011		0.009	0.008
Time taken (y hours)	12	6	4	3	2.4	2	1.7	1.5	1.3	1.2	1.1	1

Table 1.9

4. Plot the graph of y against X in Fig. 1.5.

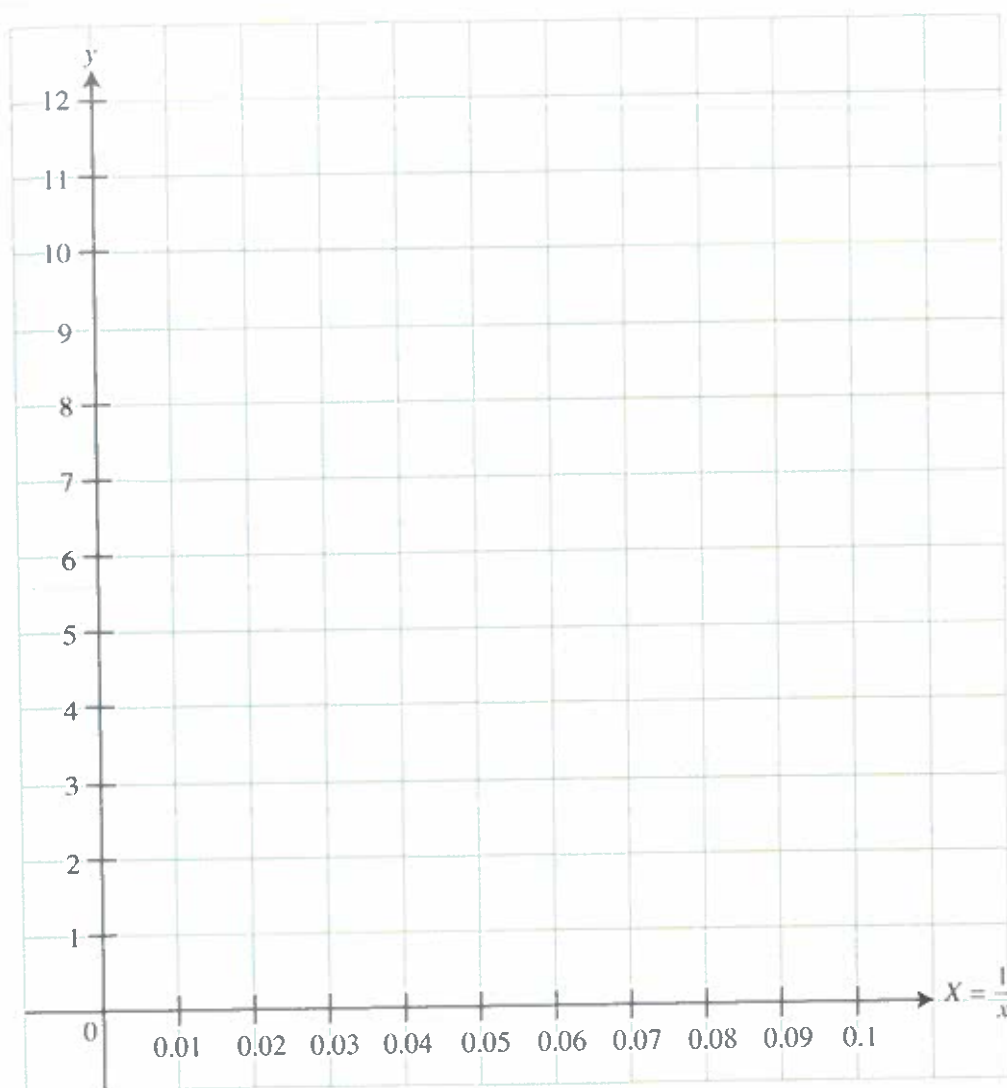


Fig. 1.5

5. Describe the graph obtained. What can we say about the relationship between y and X ?
6. Although y is inversely proportional to x , what is the relationship between y and X ?
7. Write down an equation connecting y and X . What does it tell you about the relationship between y and X ?

In general,

if y is *inversely proportional* to x , then

- $xy = k$ or $y = \frac{k}{x}$, where k is a constant and $k \neq 0$,
- the graph of y against x (and of equation $y = \frac{k}{x}$) is a hyperbola (see Fig. 1.4),
- the graph of y against $\frac{1}{x}$ is a straight line (see Fig. 1.5).



If y is inversely proportional to x , is x inversely proportional to y ? Explain your answer.

Worked Example 10

(Equation of Inverse Proportion)

If y is inversely proportional to x and $y = 3$ when $x = 4$, find

- (i) the value of y when $x = 8$,
- (ii) an equation connecting x and y ,
- (iii) the value of x when $y = 48$.

Solution:

- (i) When $x = 8$, x is doubled

$$\begin{aligned}y &= \frac{3}{2} \text{ (} y \text{ is halved)} \\&= 1\frac{1}{2}\end{aligned}$$

Alternatively,

$$x_2 y_2 = x_1 y_1$$

$$8 \times y = 4 \times 3$$

$$\begin{aligned}y &= \frac{12}{8} \\&= 1\frac{1}{2}\end{aligned}$$

- (ii) Since y is inversely proportional to x ,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 4$, $y = 3$,

$$3 = \frac{k}{4}$$

$$\therefore k = 12$$

$$\therefore y = \frac{12}{x}$$

- (iii) When $y = 48$,

$$48 = \frac{12}{x}$$

$$\begin{aligned}\therefore x &= \frac{12}{48} \\&= \frac{1}{4}\end{aligned}$$



Since y is inversely proportional to x , then $xy = k$ or $y = \frac{k}{x}$, where k is a constant and $k \neq 0$.



Raj and Ethan are going to compete in a race. Raj's average speed is twice that of Ethan's.

Ethan wants to start 10 m in front of Raj. Ethan says, 'After Raj runs 10 m, I will be 5 m in front of him. After Raj runs another 5 m, I will be 2.5 m in front of him. Thus Raj will never catch up with me.' Is Ethan correct?

This is a variation of one of Zeno's paradoxes. Zeno (490 – 430 B.C.) was a Greek philosopher.

PRACTISE NOW 10

- If y is inversely proportional to x and $y = 5$ when $x = 2$, find
 - the value of y when $x = 8$,
 - an equation connecting x and y ,
 - the value of x when $y = 10$.
- If y is inversely proportional to x and $y = 9$ when $x = 2$, find the value of y when $x = 3$.
- Given that y is inversely proportional to x , copy and complete the table.

x	0.5		2	3	
y		4	2		0.8

Worked Example 11

Problem Involving Inverse Proportion

Boyle's Law states that the volume, $V \text{ dm}^3$, of a fixed mass of gas at constant temperature is inversely proportional to its pressure, P pascals (Pa). The pressure of 1 dm^3 of a gas in an airtight container is 50 Pa. Assuming that the temperature in the container is constant, calculate the volume of the gas when its pressure is 1250 Pa.

Solution:

Since V is inversely proportional to P ,

then $V = \frac{k}{P}$, where k is a constant.

When $P = 50$, $V = 1$,

$$\begin{aligned} 1 &= \frac{k}{50} \\ \therefore k &= 50 \\ \therefore V &= \frac{50}{P} \end{aligned}$$

When $P = 1250$,

$$\begin{aligned} V &= \frac{50}{1250} \\ &= 0.04 \end{aligned}$$

$\therefore$ The volume of the gas is 0.04 dm^3 .

SIMILAR QUESTIONS

Exercise 1C Questions 3–4, 8–9, 10(a)–(b)

INFORMATION

$1 \text{ dm}^3 = 1000 \text{ cm}^3$
(dm^3 means 'cubic decimetre')

PRACTISE NOW 11

The current, I amperes (A), flowing through a wire is inversely proportional to its resistance, R ohms (Ω). Given that the current flowing through a wire with a resistance of 0.5Ω is 12 A, find

- the current flowing through the wire when its resistance is 3Ω ,
- the resistance of the wire when the current flowing through it is 3 A.

SIMILAR QUESTIONS

Exercise 1C Questions 11–12



Exercise 1C

BASIC LEVEL

- Which of the following quantities are in inverse proportion? State the assumption made in each case.
 - (a) The number of pencils Farhan buys and the total cost of the pencils.
 - (b) The number of taps filling a tank and the time taken to fill the tank.
 - (c) The number of men laying a road and the time taken to finish laying the road.
 - (d) The number of cattle to be fed and the amount of fodder.
 - (e) The number of cattle to be fed and the time taken to finish a certain amount of the fodder.

Eight men can build a bridge in 12 days. Find the time taken for 6 men to build the same bridge. State the assumption made.

If x is inversely proportional to y and $x = 40$ when $y = 5$, find

- (i) the value of x when $y = 25$,
- (ii) an equation connecting x and y ,
- (iii) the value of y when $x = 400$.

If Q is inversely proportional to P and $Q = 0.25$ when $P = 2$,

- (i) express Q in terms of P ,
- (ii) find the value of Q when $P = 5$,
- (iii) calculate the value of P when $Q = 0.2$.

INTERMEDIATE LEVEL

35 workers are employed to complete a construction project in 16 days. Before the project starts, the boss of the company is told that he needs to complete the project in 14 days. Assuming that all the workers work at the same rate, how many more workers does he need to employ in order to complete the project on time?

6. A consignment of fodder can feed 1260 cattle for 50 days. Given that all the cattle consume the fodder at the same rate, find
 - (i) the number of cattle an equal consignment of fodder can feed for 75 days,
 - (ii) the number of days an equal consignment of fodder can last if it is used to feed 1575 cattle.
7. At a sports camp, there is sufficient food for 72 athletes to last 6 days. If 18 athletes are absent from the camp, how many more days can the food last for the other athletes? State the assumption made.
8. If z is inversely proportional to x and $z = 5$ when $x = 7$, find the value of x when $z = 70$.
9. If B is inversely proportional to A and $B = 3.5$ when $A = 2$, find the value of B when $A = 1.4$.
10. For each of the following, y is inversely proportional to x . Copy and complete the tables.

(a)

x		2	2.5	3	
y	24			4	1.5

(b)

x	3	4			25
y		9	8	2.5	

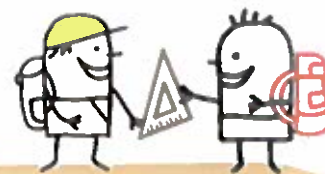
11. The frequency, f kilohertz (kHz), of a radio wave is inversely proportional to its wavelength, λ m. The frequency of a radio wave that has a wavelength of 3000 m is 100 kHz. Find
 - (i) the frequency of a radio wave that has a wavelength of 500 m,
 - (ii) the wavelength of a radio wave that has a frequency of 800 kHz.

12. The time, t hours, needed to complete a job is inversely proportional to N , where N is the number of men employed for the job. 3 men can complete the job in 8 hours.
- Find an equation connecting t and N .
 - Find the number of hours needed by 6 men to complete the job.
 - If the job is to be completed in $\frac{3}{4}$ hour, how many men need to be employed?
14. A consignment of fodder can feed 1000 sheep for 20 days. Assuming that all the sheep consume the fodder at the same rate, how many consignments of fodder are needed to feed 550 sheep for 400 days?
15. Tap A takes 6 minutes to fill a tank and Tap B takes 9 minutes to fill the same tank. Pipe C can empty the tank in 15 minutes. How long will it take to fill up the tank if the pipe is in use when both taps are turned on?
16. A contractor agrees to lay a road 3000 m long in 30 days. 50 men are employed and they work for 8 hours per day. After 20 working days, he finds that only 1200 m of the road is completed. How many more men does he need to employ in order to finish the project on time if each man now works 10 hours a day?

ADVANCED LEVEL

13. 12 glassblowers can make 12 identical vases in 9 minutes. Assuming that all the glassblowers work at the same rate, how long will it take 8 glassblowers to make 32 such vases?

1.6 Other Forms of Inverse Proportion



We have learnt that if y is inversely proportional to x , then

$$xy = k \text{ or } y = \frac{k}{x},$$

where k is a constant and $k \neq 0$.

Similarly, if y is inversely proportional to x^2 , then

$$x^2y = k \text{ or } y = \frac{k}{x^2},$$

where k is a constant and $k \neq 0$.

INFORMATION

Another way to look at this is to let $X = x^2$ such that

$$x^2y = k \text{ becomes } Xy = k$$

$$\text{or } y = \frac{k}{x^2} \text{ becomes } y = \frac{k}{X},$$

where k is a constant and $k \neq 0$.
Therefore, y is inversely proportional to X ($= x^2$).

Worked Example 12

Identifying Variables Which are Inversely Proportional to Each Other

For each of the following equations, state the two variables which are inversely proportional to each other and explain your answer.

(a) $y = \frac{2}{x^3}$

(b) $y = \frac{3}{\sqrt{x}}$

Solution:

(a) Since $y = \frac{2}{x^3}$, i.e. $x^3y = 2$ is a constant, then y is inversely proportional to x^3 .

(b) Since $y = \frac{3}{\sqrt{x}}$, i.e. $y\sqrt{x} = 3$ is a constant, then y is inversely proportional to $\sqrt{x}$.

PRACTISE NOW 12

For each of the following equations, state the two variables which are inversely proportional to each other and explain your answer.

(a) $y = \frac{4}{x^2}$

(b) $y^2 = \frac{1}{\sqrt{x}}$

(c) $y = \frac{5}{x+2}$

SIMILAR QUESTIONS

Exercise 1D Questions 3(a)–(e)

Worked Example 13

Equations of Inverse Proportion

If y is inversely proportional to $\sqrt{x}$ and $y = 6$ when $x = 4$,

- calculate the value of y when $x = 16$,
- find an equation connecting x and y ,
- find the value of x when $y = 4$.

Solution:

When $x = 16 = 4 \times 4$ (16 is 4 times of 4),

$$\begin{aligned} y &= \frac{1}{\sqrt{4}} \times 6 \quad (y \text{ is } \frac{1}{\sqrt{4}} \text{ times of } 6 \text{ since } y \text{ is inversely proportional to } \sqrt{x}) \\ &= \frac{1}{2} \times 6 \\ &= 3 \end{aligned}$$

RECALL

The square root sign $\sqrt{\quad}$ is used to denote the *positive square root* only. Thus $\sqrt{4} = 2$. However, if $x^2 = 4$, then $x = \pm\sqrt{4} = \pm 2$. If we want both the positive and the negative square roots, we need to write $\pm\sqrt{\quad}$.

- (ii) Since y is inversely proportional to $\sqrt{x}$, then $y = \frac{k}{\sqrt{x}}$, where k is a constant.

When $x = 4$, $y = 6$,

$$6 = \frac{k}{\sqrt{4}}$$

$$6 = \frac{k}{2}$$

$$\therefore k = 12$$

$$\therefore y = \frac{12}{\sqrt{x}}$$

- (iii) When $y = 4$,

$$4 = \frac{12}{\sqrt{x}}$$

$$\sqrt{x} = \frac{12}{4}$$

$$= 3$$

$$\therefore x = 9$$

PRACTISE NOW 13

- If y is inversely proportional to x^2 and $y = 2$ when $x = 4$,
 - find the value of y when $x = 8$,
 - find an equation connecting x and y ,
 - calculate the values of x when $y = 8$.
- If y is inversely proportional to $\sqrt{x}$ and $y = 6$ when $x = 9$, find the value of y when $x = 25$.
- Given that y is inversely proportional to $\sqrt{x}$, copy and complete the table.

x		1	4	16	
y	16	8			$1\frac{1}{3}$

SIMILAR QUESTIONS

Exercise 11D Questions 1–2, 4–6, 9–10

Worked Example 14

(Problem involving Another Form of Inverse Proportion)

In a computer simulation of an experiment, a drug is added to two identical flasks, each containing the same amount of a certain bacteria. The drug is allowed to react with the bacteria for various times in t hours. It is found that the amount of bacteria left, s units, is inversely proportional to $(t - 2)$ hours. In one flask, there are 6 units of bacteria left after 5 hours. Calculate the amount of bacteria left in the other flask after 7 hours.

Solution:

Since s is inversely proportional to $t - 2$,

then $s = \frac{k}{t - 2}$, where k is a constant.

When $t = 5$, $s = 6$,

$$6 = \frac{k}{5 - 2}$$

$$6 = \frac{k}{3}$$

$$\therefore k = 18$$

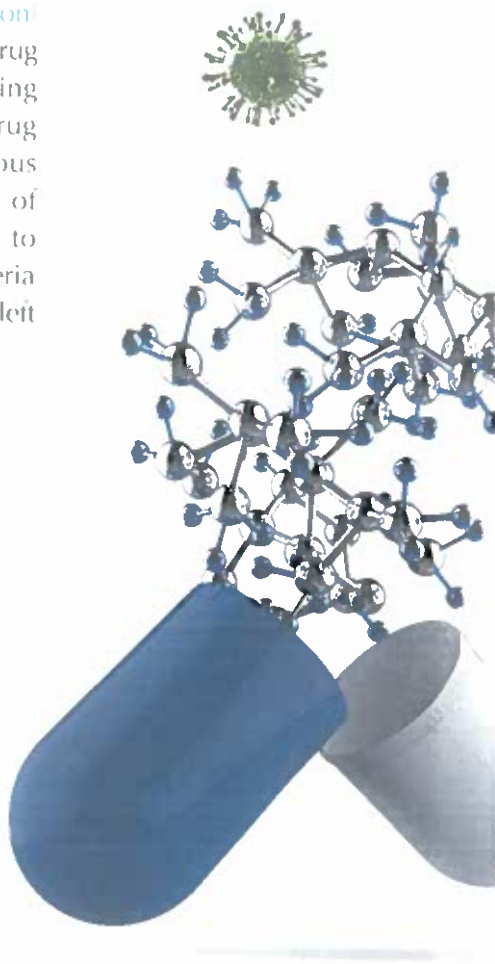
$$\therefore s = \frac{18}{t - 2}$$

When $t = 7$,

$$s = \frac{18}{7 - 2}$$

$$= 3.6$$

$\therefore$ The amount of bacteria left in the other flask after 7 hours is 3.6 units.



PRACTISE NOW 14

The force, F newtons (N), between two particles is inversely proportional to the square of the distance, d m, between them. When the particles are 2 m apart, the force between them is 10 N. Find

- the force between the particles when they are 5 m apart,
- the distance between the particles when the force between them is 25 N.

SIMILAR QUESTIONS

Exercise 1D Questions 7–8, 11



Exercise 1D

BASIC LEVEL

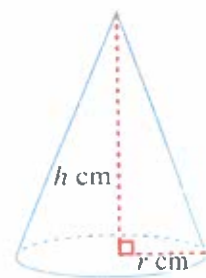
- If x is inversely proportional to y^3 and $x = 50$ when $y = 2$,
 - find the value of x when $y = 4$,
 - find an equation connecting x and y ,
 - calculate the value of y when $x = 3.2$.
- If z is inversely proportional to $\sqrt{w}$ and $z = 9$ when $w = 9$,
 - find an equation connecting w and z ,
 - find the value of z when $w = 16$,
 - calculate the value of w when $z = 3$.

INTERMEDIATE LEVEL

- For each of the following equations, state the two variables which are inversely proportional to each other and explain your answer.
 - $y = \frac{3}{x^2}$
 - $y = \frac{1}{\sqrt{x}}$
 - $y^2 = \frac{5}{x^3}$
 - $n = \frac{7}{m-1}$
 - $q = \frac{4}{(p+1)^2}$
- If z is inversely proportional to $\sqrt{x}$ and $z = 5$ when $x = 64$, find the value of z when $x = 216$.
- If q^2 is inversely proportional to $p+3$ and $q = 5$ when $p = 2$, find the values of q when $p = 17$.
- Given that t is inversely proportional to s^3 , copy and complete the table.

s	1	2	4		
t	80			0.08	0.01

- The force of repulsion, F newtons (N), between two particles is inversely proportional to the square of the distance, d m, between the particles.
 - Write down a formula connecting F and d .
 - When the particles are a certain distance apart, the force of repulsion is 20 N. Find the force when the distance is halved.
- For a fixed volume, the height, h cm, of a cone is inversely proportional to the square of the base radius, r cm. Cone A has a base radius of 6 cm and a height of 5 cm. The base radius of Cone B is 3 cm and the height of Cone C is 1.25 cm. If all the cones have the same volume, find
 - the height of Cone B,
 - the base radius of Cone C.



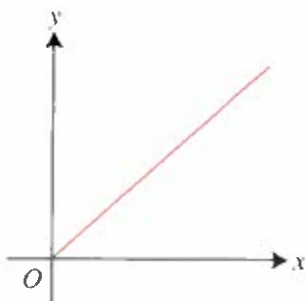
ADVANCED LEVEL

- If y is inversely proportional to $2x+1$ and the difference in the values of y when $x = 0.5$ and $x = 2$ is 0.9, find the value of y when $x = -0.25$.
- y is inversely proportional to x^2 and $y = b$ for a particular value of x . Find an expression in terms of b for y when this value of x is tripled.
- The force of attraction between two magnets is inversely proportional to the square of the distance between them. When the magnets are r cm apart, the force of attraction between them is F newtons (N). If the distance between the magnets is increased by 400%, the force of attraction between them becomes cF N. Find the value of c .



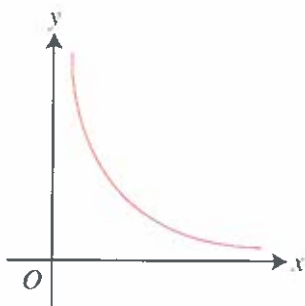
1. If y is *directly proportional* to x , then

- $\frac{y}{x} = k$ or $y = kx$, where k is a constant and $k \neq 0$,
- $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ or $\frac{x_2}{x_1} = \frac{y_2}{y_1}$,
- the graph of y against x is a straight line that passes through the origin.

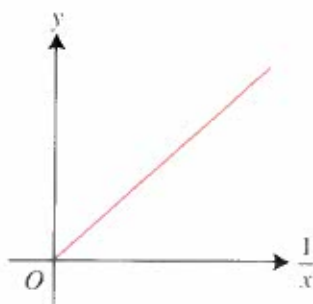


2. If y is *inversely proportional* to x , then

- $xy = k$ or $y = \frac{k}{x}$, where k is a constant and $k \neq 0$,
- $\frac{y_2}{y_1} = \frac{x_1}{x_2}$ or $x_1 y_1 = x_2 y_2$,
- the graph of y against x is a hyperbola,



- the graph of y against $\frac{1}{x}$ is a straight line that starts from the origin ($x \neq 0$).



Review Exercise 1



- If y is directly proportional to x and $y = 6$ when $x = 2$,
 - express y in terms of x ,
 - find the value of y when $x = 11$,
 - calculate the value of x when $y = 12$,
 - draw the graph of y against x .
- If A is directly proportional to B and $A = 1\frac{2}{3}$ when $B = \frac{5}{6}$, find
 - the value of A when $B = \frac{1}{3}$,
 - the value of B when $A = 7\frac{1}{2}$.
- If y is directly proportional to x^3 and $y = 108$ when $x = 3$,
 - find an equation connecting x and y ,
 - find the value of y when $x = 7$,
 - calculate the value of x when $y = 4000$,
 - draw the graph of y against x^3 .
- If n is directly proportional to m^2 and $n = 9.375$ when $m = 2.5$, find
 - the value of n when $m = 3$,
 - the values of m when $n = 181.5$.
- If t is directly proportional to $\sqrt{s}$ and $t = 4$ when $s = 64$, find
 - the value of t when $s = 125$,
 - the value of s when $t = 2$.
- If y is inversely proportional to x and $y = 4$ when $x = 3$,
 - express y in terms of x ,
 - find the value of y when $x = 6$,
 - calculate the value of x when $y = 24$.
- If q is inversely proportional to p^2 and $q = 3$ when $p = 5$, find
 - an equation connecting p and q ,
 - the value of q when $p = 10$,
 - the negative value of p when $q = \frac{1}{3}$.

- If z is inversely proportional to $w + 3$ and $z = 4$ when $w = 3$, find
 - the value of z when $w = 9$,
 - the value of w when $z = 2.4$.

- Given that y is inversely proportional to $2x^2$, copy and complete the table.

x	0.2	0.5			2
y			1.5	0.96	0.375

- The total monthly charges, \$ C , for a fixed phone line consists of a fixed amount of \$9.81 and a variable amount which depends on the usage. For every minute used, \$0.086 is charged.
 - If the duration of usage is 300 minutes, find the total monthly charges for the fixed phone line.
 - If the total monthly charges for the fixed phone line are \$20.56, calculate the duration of usage.
 - Write down a formula connecting C and n , where n is the number of minutes of usage. Hence, state the two variables which are directly proportional to each other.
- The gravitational potential energy, G joules (J), of an object is directly proportional to its height, h m, above the surface of the Earth. When the object is at a height of 40 m above the surface of the Earth, its gravitational potential energy is 2200 J. Find
 - an equation connecting G and h ,
 - the gravitational potential energy of the object when it is at a height of 22 m above the surface of the Earth,
 - the height of the object above the surface of the Earth when it has a gravitational potential energy of 3025 J.

12. Kate makes a donation to a charitable organisation on a monthly basis. Her monthly donation is directly proportional to the square of her monthly savings. If she saves \$900 and \$1200 in January and February respectively, her donation increases by \$35 from January to February. Find the amount of money she donates to the charitable organisation in each of the two months.
13. Boyle's Law states that the pressure, P pascals (Pa), of a fixed mass of gas at constant temperature is inversely proportional to its volume, $V \text{ dm}^3$. The pressure of 4000 dm^3 of a gas in an airtight container is 250 Pa. Assuming that the temperature in the container is constant, find
- the pressure of the gas when its volume is 5000 dm^3 ,
 - the volume of the gas when its pressure is 125 Pa.
14. 5 men are hired to complete a job. If one more man is hired, the job can be completed 8 days earlier. Assuming that all the men work at the same rate, how many more men should be hired so that the job can be completed 28 days earlier?



Challenge Yourself

- If A is directly proportional to C and B is directly proportional to C , prove that each of the following is directly proportional to C .
 - $A + B$
 - $A - B$
 - $\sqrt{AB}$
- It is given that z is directly proportional to x^2 and inversely proportional to $\sqrt{y}$.
 - Write down an equation connecting x , y and z .
 - If $z = 16$ when $x = 2$ and $y = 9$, find the value of z when $x = 5$ and $y = 4$.
- The time, T days, needed to paint some buildings is directly proportional to the number of buildings, B , that need to be painted and inversely proportional to the number of painters employed, P . 18 painters can paint 3 buildings in 20 days.
 - Find an equation connecting T , B and P .
 - Find the number of days needed by 16 painters to paint 4 buildings.
 - If 10 buildings are to be painted in 24 days, how many painters need to be employed?